Petri Nets

- Part of this discussion is based on the paper
  - *Petri Nets: Properties, Analysis and Applications*

- Petri Nets
  - graphical and mathematical modeling tool
  - tool for describing systems characterized as being:
    » concurrent, asynchronous, distributed, parallel, nondeterministic
      and/or stochastic
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- **History**
  - **1962:** Carl Adam Petri’s submitted his dissertation at the Uni. Darmstadt, Germany
  - **1970:** early development was published by A.W. Host and in the records of the 1970 Project MAC Conference on Concurrent Systems and Parallel Computation
  - **1970-75:** Computation Structure Group and MIT was most active
  - **1975:** conference on Petri Nets and Related Methods at MIT
  - **1979:** 135 researchers assembled in Hamburg, Germany, for 2-week advanced course on General Net Theory of Processes and Systems
  - **1980:** first European Workshop on Applications and Theory of Petri Nets, Strasbourg, France.
    - check out Murata’s paper for the extensive literature discussion
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◆ General:
  - directed, weighted, bipartite graph
  - two kinds of notes (Places P, Transitions T)
  - arcs from P to T or from T to P
  - arcs have integer weights
  - non-negative Place weights are called tokens
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- A Petri Net is a 5-touple $PN = \{P, T, A, W, M_0\}$
- Place Set $P = \{p_1, p_2, ..., p_m\}$
  - finite set of places
  - condition = place
  - one condition or set of atomic conditions
  - symbol

- Transition Set $T = \{t_1, t_2, ..., t_n\}$
  - finite set of transitions
  - action = transition
  - one action or set of atomic transitions
  - symbol
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◆ Arc Set $A \subseteq (P \times T) \cup (T \times P)$
  - set of directed arcs
  - edge of graph = arc
  - symbol $\rightarrow$

◆ Weight Function $W = A \rightarrow \{1, 2, 3, \ldots\}$
  - weights are associated with arcs

◆ Initial Marking $M_0 = P \rightarrow \{0, 1, 2, \ldots\}$
  - the initial assignment of tokens to places
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◆ example

[Diagram of Petri Nets]
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- Dynamic Behavior
  - during simulation of a petri net the state of the net may change
  - change of state:
    » transitions can be enabled
    » enabled transitions may fire
    » firing transition changes the marking of the net
    » the marking is the “snap-shot” of all the tokens
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❖ Firing rules
  – A transition \( T \) is said to be *enabled* if each input place \( P \) is marked with at least \( W(P,T) \) tokens
    » \( W(P,T) \) is the weight of the arc from \( P \) to \( T \)
  – An enabled transition may or may not fire (depending on whether or not the event actually takes place).
  – A *firing* of an enabled transition \( T \) removes \( W(P,T) \) tokens from each input place \( P \) of \( T \), and adds \( W(T,P) \) tokens to each output place \( P \) of \( T \)
    » \( W(T,P) \) is the weight of the arc from \( T \) to \( P \)

  – Common misconception: When a transition fires, it does **not** move tokens
    » i.e. the number of tokens in the system is not necessarily constant
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- Example: assume the following initial marking
  - Only one transition is enabled, i.e. $t_2$
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- Now several transitions are enabled, i.e. $t_1, t_3$ and $t_5$
- if $t_1$ fires first
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- if $t_3$ fires first
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- if $t_5$ fires first
- $t_3$ and $t_5$ are said to be in conflict
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- what could this Petri net represent?
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- Marking: Number and placement of tokens
  - let $m_i = \# \text{ of tokens in place } p_i$
  - then marking
    $$M = \{m_1, m_2, \ldots, m_n\}$$
  - marking -- system state
  - Advantage: economy of model
    » e.g. assume net with 6 places
      ■ we limit each place to maximal 1 token
      ■ then there are $2^6$ possible markings
      ■ $\Rightarrow$ 64 states
      ■ thus Petri Nets are a lot smaller than state diagrams, i.e. Markov chains
**Petri Nets**

- **Firing rules**
  - transition 1, 3 and 4 are enabled
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- Firing rules
  - transition 4 fires
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- Firing rules
  - transition 1 fires
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- Firing rules
  - transition 3 fires