Petri Nets

- Part of this discussion is based on the paper
  - *Petri Nets: Properties, Analysis and Applications*

Petri Nets

- graphical and mathematical modeling tool
- tool for describing systems characterized as being:
  - concurrent, asynchronous, distributed, parallel, nondeterministic and/or stochastic

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Petri Nets

- History
  - 1962: Carl Adam Petri’s submitted his dissertation at the Uni. Darmstadt, Germany
  - 1970: early development was published by A.W. Host and in the records of the 1970 Project MAC Conference on Concurrent Systems and Parallel Computation
  - 1970-75: Computation Structure Group and MIT was most active
  - 1975: conference on Petri Nets and Related Methods at MIT
  - 1979: 135 researchers assembled in Hamburg, Germany, for 2-week advanced course on General Net Theory of Processes and Systems
    - check out Murata’s paper for the extensive literature discussion
Petri Nets

* General:
  - directed, weighted, bipartite graph
  - two kinds of notes (Places P, Transitions T)
  - arcs from P to T or from T to P
  - arcs have integer weights
  - non-negative Place weights are called tokens

Petri Nets

* A Petri Net is a 5-touple PN={P,T,A,W,M0}

* Place Set P = \{p_1, p_2, ..., p_m\}
  - finite set of places
  - condition = place
  - one condition or set of atomic conditions
  - symbol

* Transition Set T = \{t_1, t_2, ..., t_n\}
  - finite set of transitions
  - action = transition
  - one action or set of atomic transitions
  - symbol
Petri Nets

- **Arc** Set $A \subseteq (P \times T) \cup (T \times P)$
  - set of directed arcs
  - edge of graph = arc
  - symbol $\rightarrow$

- **Weight Function** $W = A \rightarrow \{1,2,3,\ldots\}$
  - weights are associated with arcs

- **Initial Marking** $M_0 = P \rightarrow \{0,1,2,\ldots\}$
  - the initial assignment of tokens to places

Petri Nets

- example

![Petri Net Diagrams](image-url)
Petri Nets

◆ Dynamic Behavior
  – during simulation of a petri net the state of the net may change
  – change of state:
    » transitions can be enabled
    » enabled transitions may fire
    » firing transition changes the marking of the net
    » the marking is the “snap-shot” of all the tokens

Petri Nets

◆ Firing rules
  – A transition $T$ is said to be enabled if each input place $P$ is marked with at least $W(P,T)$ tokens
    » $W(P,T)$ is the weight of the arc from $P$ to $T$
  – An enabled transition may or may not fire (depending on whether or not the event actually takes place).
  – A firing of an enabled transition $T$ removes $W(P,T)$ tokens from each input place $P$ of $T$, and adds $W(T,P)$ tokens to each output place $P$ of $T$
    » $W(T,P)$ is the weight of the arc from $T$ to $P$
  – Common misconception: When a transition fires, it does not move tokens
    » i.e. the number of tokens in the system is not necessarily constant
**Petri Nets**

- Example: assume the following initial marking
  - Only one transition is enabled, i.e. $t_2$

- Now several transitions are enabled, i.e. $t_1 \ t_3 \text{ and } t_5$
  - if $t_1$ fires first
**Petri Nets**

- if $t_3$ fires first

![Diagram of Petri Nets]

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- if $t_5$ fires first
  - $t_3$ and $t_5$ are said to be in conflict

![Diagram of Petri Nets]
Petri Nets

- what could this Petri net represent?

![Petri Net Diagram]

- Marking: Number and placement of tokens
  - let \( m_i \) = # of tokens in place \( p_i \)
  - then marking
    \[ M = \{ m_1, m_2, \ldots, m_n \} \]
  - marking -- system state
  - Advantage: economy of model
    - e.g. assume net with 6 places
      - we limit each place to maximal 1 token
      - then there are \( 2^6 \) possible markings
      - \( \Rightarrow \) 64 states
      - thus Petri Nets are a lot smaller than state diagrams, i.e. Markov chains
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- Firing rules
  - transition 1,3 and 4 are enabled

![Petri Net Diagram](image1)

- Firing rules
  - transition 4 fires

![Petri Net Diagram](image2)
Petri Nets

- Firing rules
  - transition 1 fires

Petri Nets

- Firing rules
  - transition 3 fires