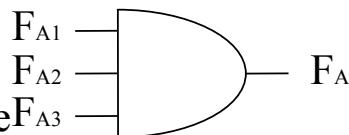


# Fault Trees

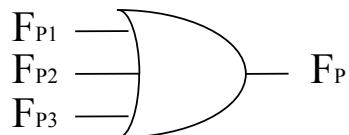
- ◆ Fault Trees
  - dual of Reliability Block Diagram
  - logic failure diagram
  - think in terms of logic where
    - » 0 = operating, 1 = failed
- ◆ AND Gate
  - all inputs must fail for the gate to fail
- ◆ OR Gate
  - any input failure causes the gate to fail
- ◆ k-of-n Gate
  - k or more input failures cause gate to fail

## e.g. Triplex Bus Guardian

- ◆ Active mode
  - M<sub>1</sub> and M<sub>2</sub> and M<sub>3</sub> fail =>
  - AND Gate
- ◆ Passive Mode

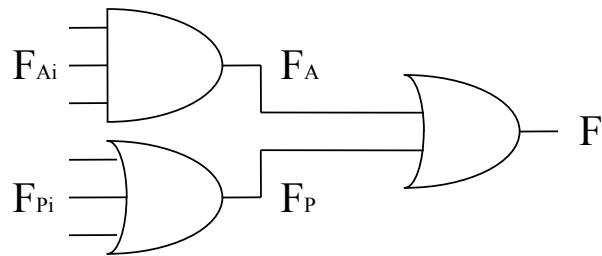


- “cutoff” with any single unit failure =>
- OR Gate



## *e.g. Triplex Bus Guardian*

- ◆ Total Failure
  - caused by either active or passive mode



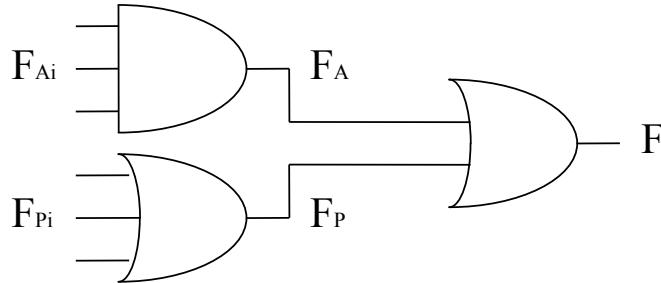
## *e.g. Triplex Bus Guardian*

- ◆ How can one use the fault tree effectively to isolate those parts of the system that need reliability considerations?

## e.g. Triplex Bus Guardian

- ◆ Combined fault model

$$Q(1000h) = 0.9851243 \cdot 10^{-6}$$



$$Q(1000h) = 0.295545 \cdot 10^{-1}$$

## Examples

- ◆ Simple Passive TMR (no diagnosis)
  - RBD = (2 of 3): 2 operable => System operable
  - F-Tree = (2 of 3): 2 failed => System failed
- ◆ Simple TMR with *Benign* failures
  - RBD = (1 of 3): 1 operable => System operable
  - F-Tree = (3 of 3): 3 failed => System failed
- ◆ Summary
  - Parallel => AND
  - Series => OR
  - K-of-N => (n-k+1 of n)

# *SHARPE*

- ◆ SYMBOLIC HIERARCHICAL AUTOMATED RELIABILITY AND PERFORMANCE EVALUATOR
- ◆ SHARPE provides a specification language and analysis algorithms for the following model types:
  - reliability block diagrams
  - fault trees
  - reliability graphs
  - series-parallel acyclic directed graphs
  - product-form queuing networks
  - Markov and semi-Markov chains
  - generalized stochastic Petri nets
- note: we are using the latest version of SHARPE and there could be changes that are not reflected in this handout.

# *SHARPE*

- ◆ Read the “SHARPE LANGUAGE DESCRIPTION” for details. (The material below is directly adopted from there).
- ◆ Basic language components
  - **comments** ‘\*’ indicates a comment line
  - **echo** *anytext* writes *anytext* to the output
  - **include** *filename* takes inputs from the file indicated
    - names: command **info constants** on the current version under solaris indicates:

```
size of input line: 512
maximum number of intervals for eval: 100
number of significant characters in words: 30
maximum number of phases in a phased-mission system (pms): 100
```

# *SHARPE*

## ◆ Basic language components

- A *word* is a sequence of characters (no white space, commas, semicolons, ( ) \ \$).
- A *subword* is a string \$*n* or \$(*expression*).
- » *n* is a single letter
  - » any expression can be used within the parenthesis
- *evaluated word* provides limited means of index
  - » e.g. let i =4 and j=5, then the evaluated word \$(i)A-\$(j-i)B-\$j evaluates to the component name: 4A-1B-5
- length of words: as before, use **info constants**

# *SHARPE*

## ◆ Basic language components (cont.)

- Arithmetic Expressions
  - » addition +
  - » subtraction -
  - » negation -
  - » division /
  - » multiplication \*
  - » exponentiation ^
  - » “power of e” ^

# *SHARPE*

## ◆ Basic language components (cont.)

### - Arithmetic Expressions

- » evaluated from left to right
- » examples:

```
360 / (360 + (k-1) * x)
k * x * (1 - c(x, k))
3 * lambda * prob(second-fault, recovered; 2*lambda)
^(k-1) * x * tau
delta / ((k-1) * x) * (1 - ^(-(k-1) * x / delta))
sum (i,1,n,sum(j,1,i,j))
sum (i,1,n,prob(m,\$(i)-\$(j)))
```

# *SHARPE*

## ◆ Basic language components (cont.)

- simple variables (*simple\_var*), a *name* that must be bound to a value

**bind** *simple\_var expression*

**bind**

*<simple\_var expression>*

**end**

- function

**func** *function\_name (param\_list) expression*

- defined variable (is the same as a function without arguments)

**var** *defined\_var expression*

# SHARPE

## ◆ Basic language components (cont.)

### - Scope of names and words

- » simple/defined variables, function names, model names, exp. polynomial names are global
- » component names are local to the model in which they appear
- » parameter names are local to their system, function etc.
- » index of a loop is local to the loop
- » index of a sum function is local to the function

### - parameter list (*param\_list*)

*name, name, ... , name*

### - argument list (*arg\_list*)

*expression, expression, ... , expression*

# cdf $F(t)$

- $F(t)$ , is the cumulative distribution function for the failure probability of the component.
- $F(t)$  is the probability that component  $i$  has failed by time  $t$ . SHARPE can compute the system failure distribution function and the mean and variance of the function. It can be asked to evaluate the function at particular values of  $t$ .

$$F(t) = \sum_{j=1}^k a_j t^{k_j} e^{b_j t}$$

The diagram shows the formula for the cumulative distribution function  $F(t)$  as a sum of terms. Each term consists of a coefficient  $a_j$ , a power of  $t$  ( $t^{k_j}$ ), and an exponential factor ( $e^{b_j t}$ ). Three boxes below the formula have arrows pointing to these components:

- The first box contains "real or complex" and points to the coefficient  $a_j$ .
- The second box contains "non negative integer" and points to the exponent  $k_j$ .
- The third box contains "real or complex" and points to the coefficient  $b_j$ .

# *SHARPE Command Language*

- ◆ Model Definition
- ◆ Binding Values to Variables
- ◆ Calling Functions
- ◆ Printing (with built-in functions)
  
- ◆ silly details
  - \* comment
  - \ line continuation
  - names are truncated to 14 characters

## *Reliability Block Diagram (RBD)*

- ◆ block
  - ◆ comp
  - ◆ parallel
  - ◆ series
  - ◆ kofn
- SHARPE manual page 22

# Fault Trees

- ◆ ftree
  - ◆ basic
  - ◆ repeat
  - ◆ transfer
  - ◆ and
  - ◆ or
  - ◆ kofn
- SHAPRE manual page 23

# Calling Functions + Printing

- ◆ See Section B.5 of the SHARPE manual for details
- ◆ **cdf** (*system\_name* {,*state\_eword*} {;*arg\_list*}
  - all models except irreducible chain or gspn.
  - *state\_eword* used only in markov chains
  - *arg\_list* matches # of parameters in system definition
    - prints cdf, mean, variance
- ◆ **expr** *expression* {,*expression...*}
  - sharpe prints the value of the *expressions*
  - e.g.
    - » expr(lambda)
    - » expr(lam\_a + lam\_b)

# *Calling Functions + Printing*

- ◆ **eval**(*system\_name* {;*state\_eword*} {;*arg\_list*} *lhi* {*function*})
  - same as for cdf
  - *lhi* = low, high, increment are all expressions
  - *function* is cdf (default) or any other function
    - e.g. generate F(t) for t specified and print result
    - for Markov chain
      - » argument {;*state*} allows to evaluate the probability of being in the state specified.
  - *lhi* low, high, increment e.g.
    - » 100 200 25
    - » evaluate F(t) at t = 100, 125, 150, 175 and 200 hours

# *Calling Functions + Printing*

- ◆ \* comment
- ◆ **echo** *string*
  - echoes *string* to the output
  - e.g. echo this is just a test
- ◆ **var** *var\_name* *expression*
  - defines variable *var\_name*
  - assigns the value in *expression*
  - does not print

# *Calling Functions + Printing*

- ◆ **value**(*t ;system\_name {,state\_eword} {;arg\_list}*)
  - calculates value of cdf at time *t*
  - does not print
  - use on right hand side of **var**
  - e.g.
    - » var Fat1000 value(1000;bus\_gd\_pass)
    - » expr Fat1000
- ◆ **format constant**
  - number of digits after decimal point to be printed

# *Calling Functions + Printing*

- ◆ Values of “epsilon”
  - user-controlled epsilon: small values that determine when algorithms have converged or two floating point numbers are equal

**epsilon** *epsilon\_id expression*

- **basic** determines when two floats are even
- **result** determines when a printed result is considered to be zero
- e.g.
  - » epsilon basic 1.0\*10^-11
  - » epsilon results 1.0\*10^-10

# *Bus Guardian (Passive)*

```
* SYSTEM: TRIPLEX BUS GUARDIAN -- PASSIVE FAILURE MODE
* MODEL: RELIABILITY BLOCK DIAGRAM
* -- Model Definition: block name, components, connectivity --
*
block bus_gd_pas
comp z exp(lampas)
series z3 z z z
end

* -- Bind Values to Variable Names --
*
bind
lampas 1.0*10^-5
end

* -- Calculate CDF for System Failure --
*
cdf(bus_gd_pas)

* -- Evaluate CDF at Specified Points --
eval(bus_gd_pas) 9 11 1
eval(bus_gd_pas) 90 110 10
eval(bus_gd_pas) 900 1100 100
*
var Qat1000 value(1000; bus_gd_pas)
expr Qat1000
end
```

# *Bus Guardian (Passive)*

CDF for system bus\_gd\_pas:

$$1.0000e+00 t(0) \exp(0.0000e+00 t) + -1.0000e+00 t(0) \exp(-3.0000e-05 t)$$

mean: 3.3333e+04

variance: 1.1111e+09

---

system	bus_gd_pas	t	F(t)
9.0000 e+00	2.6996 e-04	1.0000 e+01	2.9996 e-04
1.1000 e+01	3.2995 e-04		

system	bus_gd_pas	t	F(t)
9.0000 e+01	2.6964 e-03	1.0000 e+02	2.9955 e-03
1.1000 e+02	3.2946 e-03		

---

system	bus_gd_pas	t	F(t)
9.0000 e+02	2.6639 e-02	1.0000 e+03	2.9554 e-02
1.1000 e+03	3.2461 e-02		

Qat1000: 2.9554e-02

# Bus Guardian (Active)

```

* SYSTEM: TRIPLEX BUS GUARDIAN -- ACTIVE FAILURE MODE
* MODEL: RELIABILITY BLOCK DIAGRAM
* -- Model Definition: block name, components, connectivity --
*
block bus_gd_act
comp z exp(lamact)
parallel z3 z z z
end

* Bind Values to Variable Names
*
bind
lamact 1.0*10^-5
end

```

---

```

* -- Calculate CDF for System Failure
*
cdf(bus_gd_act)

* -- Evaluate CDF at Specified Points
eval (bus_gd_act) 9 11 1
eval (bus_gd_act) 90 110 10
eval (bus_gd_act) 900 1100 100
*
var Qat1000 value(1000; bus_gd_act)
expr Qat1000
end

```

# Bus Guardian (Active)

CDF for system bus\_gd\_act:

$$\begin{aligned}
 & 1.0000e+00 t(0) \exp(0.0000e+00 t) \\
 & + -3.0000e+00 t(0) \exp(-1.0000e-05 t) \\
 & + 3.0000e+00 t(0) \exp(-2.0000e-05 t) \\
 & + -1.0000e+00 t(0) \exp(-3.0000e-05 t)
 \end{aligned}$$

mean: 1.8333e+05

variance: 1.3611e+10

---

system bus_gd_act	
t	F(t)

9.0000 e+00	0.0000 e+00
1.0000 e+01	0.0000 e+00
1.1000 e+01	0.0000 e+00

system bus_gd_act	
t	F(t)

9.0000 e+01	0.0000 e+00
1.0000 e+02	0.0000 e+00
1.1000 e+02	1.3288 e-09

---

system bus_gd_act	
t	F(t)

9.0000 e+02	7.1923 e-07
1.0000 e+03	9.8512 e-07
1.1000 e+03	1.3092 e-06

Qat1000: 9.8512e-07