Markov Models of Typical Systems

- Absorbing State
  - Simple Triplex System with fail-stop behavior
  - Assumptions:
    - no repair
    - perfect fault coverage
      - recall: fault coverage is a measure of the systems ability to detect faults and recover
    - homogeneous components
    - independent failures

Markov Models of Typical Systems

- System with Repair
  - Simple Triplex System with fail-stop behavior and repair
  - Same assumptions as previous example

Markov Models of Typical Systems

- Fault Coverage
  - Simple Triplex System
  - Fault Coverage
    - » C = probability that fault is properly handled
    - » C_i has major impact on reliability

Markov Models of Typical Systems

- Multiple Faults
  - Simple Triplex System with double fault

$t_r \equiv$ recovery time from fault

$\rho \equiv \frac{1}{t_r}$ recovery rate

Markov Models of Typical Systems

– Simple Triplex System with double fault (cont.)

Markov Models of Typical Systems

- Different λ's
  - e.g. hot + cold spares
    - typical MTTF(cold) = 10 MTTF(hot)
  - notation: h.c
    - h = number of hots
    - c = number of colds
  - assume perfect coverage
  - assume switching mechanism

Markov Models of Typical Systems

- Passive TMR with 2 Failure Modes
  - fail passive => processor just disconnects
  - notation: n.f
    - n = number of non-faulty processors running
    - f = number of faulty processors running
  - assume different fail rates
    - failure mode 1: benign fail rate λ_{stop}
    - failure mode 2: single non-benign fail rate λ_{err}

Markov Models of Typical Systems

- Passive TMR with 2 Failure Modes (cont.)

Sharpe & Markov Chains

Here we use SHARPE to determine the unreliabilities.

The main slide of interest is the last one that contains the probabilities of being in the specific states.

Why is this interesting? Well...

Sharpe & Markov Chains

- SHARPE extracts model and analysis type:
  - Cyclic vs. Acyclic Model
  - Steady-State vs. Transient Analysis

```
markov model_name
{param_list}
    from   to   transition_rate
    <name name expression>
end
    initial state probabilities
    <name expression>
end
```

Markov Model

- P(0) = 0 is assumed for any state not listed in initialization
- The sum of all P(0) must be 1
  - Beware of round-off errors
- Initial state probabilities section may be left empty if:
  - Acyclic model with only 1 source state
    - Assumes P(0) = 1 for that state
  - Irreducible model Steady State analysis
    - in this case initial conditions are irrelevant
- Advice: Always specify initial probabilities

Useful functions

- tvalue (t; model_name, state; arg_list)
  - Gives Transient Probabilities at time *t*.
  - If no state is given:
    - computes transient prob. of being in an absorbing state at time *t*
    - there can be more than one absorbing state
      - => prob. of being in any absorbing state.
  - If state is given:
    - computes transient prob. of being in that state at time *t*

Useful functions

- prob (model_name, state {;arg_list})
  - Gives Steady State Probabilities (no time param)
  - Note: state parameter is not optional
  - With absorbing states this computes the steady state probability of ever visiting a specific state
  - If no absorbing states exist (irreducible chain), the steady state probability of being in a specified state is computed

Interpretation of F(t)

- If a non-absorbing state is specified:
  - F(t) is the transient or steady state CDF for that state
- If an absorbing state is specified:
  - F(t) is the CDF to absorbing by that state
  - absorbing state normally indicates a specific failure mode
- If no state is specified:
  - F(t) is the CDF to include all absorbing states
  - i.e. it is the sum of all CDFs of individual absorbing states
  - e.g. indicating system failure

TMR example

```
* SYSTEM: TMR_2MODE -- PASSIVE TMR WITH: FAIL-STOP AND FAIL-ACTIVE MODES
* MODELS: MARKOV (ACYCLIC)
* STATE NOTATION: "N.F" WHERE:
*               N == NUMBER OF NON-FAULTY PROCESSORS RUNNING.
*               F == NUMBER OF FAULTY PROCESSORS STILL RUNNING.
*
*---------------------------- MODEL DEFINITIONS
MARKOV tmr_2mode
 *
 3.0 2.0   3*LAMstop
 3.0 2.1   3*LAMerr
 *
 2.0 1.0   2*LAMstop
 2.0 1.1   2*LAMerr
 *
 2.1 2.0   1*LAMstop
 2.1 1.1   2*LAMstop
 2.1 1.2   2*LAMerr
 *
 1.0 0.0   1*LAMstop
 1.0 0.1   1*LAMerr
END
*---------------------------- INITIAL CONDITIONS (START IN 3.0)
 3.0 1.00
END
```

TMR example

```
*---------------------------- PARAMETER BINDING
BIND
  LAMBDA   1*10^-4
  LAMstop  0.9*LAMBDA
  LAMerr   0.1*LAMBDA
END
*---------------------------- ANALYSES AND EVALUATIONS
cdf (tmr_2mode)
cdf (tmr_2mode,0.0)

var fail01  value(100.0;tmr_2mode,0.1)
var fail11  value(100.0;tmr_2mode,1.1)
var fail12  value(100.0;tmr_2mode,1.2)
var failrun   fail01 + fail11 + fail12
var failstop  value(100.0;tmr_2mode,0.0)
var failall   failrun + failstop

expr fail01
expr fail11
expr fail12
expr failrun
expr failstop
expr failall
END
```

TMR example

CDF for system tmr_2mode:

```
   1.0000e+00 t(  0) exp( 0.0000e+00 t)
 + -2.5579e+00 t(  0) exp(-1.0000e-04 t)
 +  2.4000e+00 t(  0) exp(-2.0000e-04 t)
 + -2.8421e+00 t(  0) exp(-2.9000e-04 t)
 +  2.0000e+00 t(  0) exp(-3.0000e-04 t)
```

mean: 1.6713e+04
variance: 1.3541e+08

----
information about system tmr_2mode node 0.0

probability of entering node: 7.5414e-01

conditional CDF for time of reaching this absorbing state

```
   1.0000e+00 t(  0) exp( 0.0000e+00 t)
 + -3.0526e+00 t(  0) exp(-1.0000e-04 t)
 +  3.2222e+00 t(  0) exp(-2.0000e-04 t)
 + -1.1696e+00 t(  0) exp(-2.9000e-04 t)
```

mean: 1.8448e+04
variance: 1.3689e+08

fail01: 7.9815e-08

----
fail11 : 5.3038e-05

----
fail12 : 2.9416e-06

----
failrun : 5.6059e-05

----
failstop: 7.1833e-07

----
failall: 5.6778e-05