

Petri Nets

- ◆ Part of this discussion is based on the paper
 - *Petri Nets: Properties, Analysis and Applications*
 - by Tadao Murata, Proc. IEEE, Vol. 77, No. 4, April 1989.

- ◆ Petri Nets
 - graphical and mathematical modeling tool
 - tool for describing systems characterized as being:
 - » concurrent, asynchronous, distributed, parallel, nondeterministic and/or stochastic



Petri Nets

- ◆ History
 - **1962:** Carl Adam Petri's submitted his dissertation at the Uni. Darmstadt, Germany
 - **1970:** early development was published by A.W. Host and in the records of the 1970 Project MAC Conference on Concurrent Systems and Parallel Computation
 - **1970-75:** Computation Structure Group and MIT was most active
 - **1975:** conference on Petri Nets and Related Methods at MIT
 - **1979:** 135 researchers assembled in Hamburg, Germany, for 2-week advanced course on General Net Theory of Processes and Systems
 - **1980:** first European Workshop on Applications and Theory of Petri Nets, Strasbourg, France.
 - check out Murata's paper for the extensive literature discussion

Petri Nets

- ◆ General:
 - directed, weighted, bipartite graph
 - two kinds of nodes (Places P, Transitions T)
 - arcs from P to T or from T to P
 - arcs have integer weights
 - non-negative Place weights are called tokens

Petri Nets

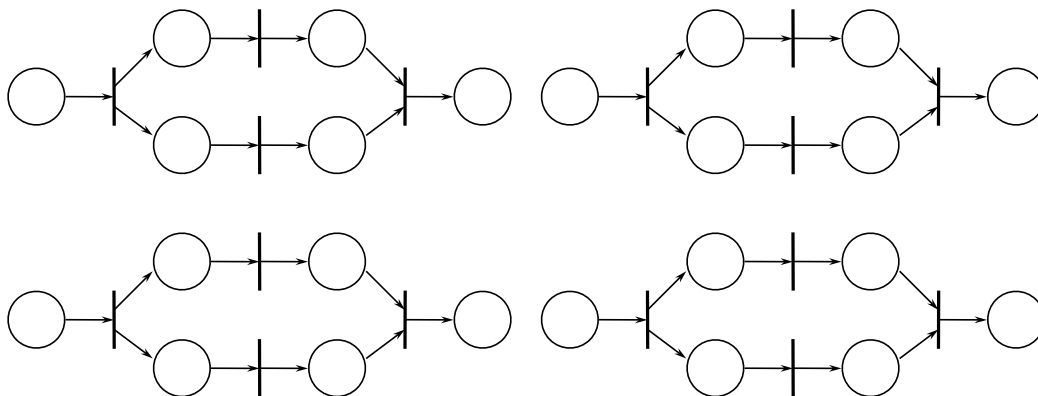
- ◆ A Petri Net is a 5-tuple $PN = \{P, T, A, W, M_0\}$
- ◆ Place Set $P = \{p_1, p_2, \dots, p_m\}$
 - finite set of places
 - condition = place
 - one condition or set of atomic conditions
 - symbol 
- ◆ Transition Set $T = \{t_1, t_2, \dots, t_n\}$
 - finite set of transitions
 - action = transition
 - one action or set of atomic transitions
 - symbol 

Petri Nets

- ◆ Arc Set $A \subseteq (P \times T) \cup (T \times P)$
 - set of directed arcs
 - edge of graph = arc
 - symbol \longrightarrow
- ◆ Weight Function $W = A \rightarrow \{1,2,3,\dots\}$
 - weights are associated with arcs
- ◆ Initial Marking $M_0 = P \rightarrow \{0,1,2,\dots\}$
 - the initial assignment of tokens to places

Petri Nets

- ◆ example



Petri Nets

◆ Dynamic Behavior

- during simulation of a petri net the state of the net may change
- change of state:
 - » transitions can be enabled
 - » enabled transitions may fire
 - » firing transition changes the marking of the net
 - » the marking is the “snap-shot” of all the tokens

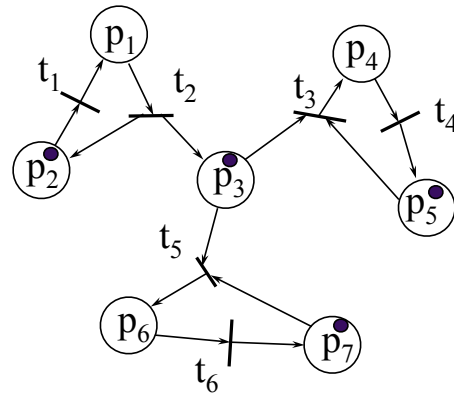
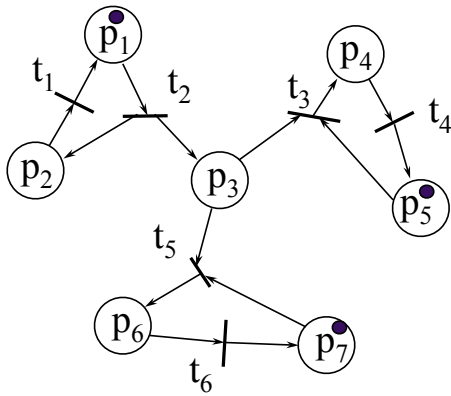
Petri Nets

◆ Firing rules

- A transition T is said to be *enabled* if each input place P is marked with at least $W(P,T)$ tokens
 - » $W(P,T)$ is the weight of the arc from P to T
- An enabled transition may or may not fire (depending on whether or not the event actually takes place).
- A *firing* of an enabled transition T removes $W(P,T)$ tokens from each input place P of T , and adds $W(T,P)$ tokens to each output place P of T
 - » $W(T,P)$ is the weight of the arc from T to P
- Common misconception: When a transition fires, it does **not** move tokens
 - » i.e. the number of tokens in the system is not necessarily constant

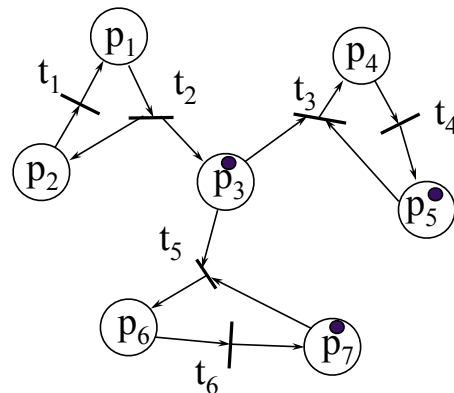
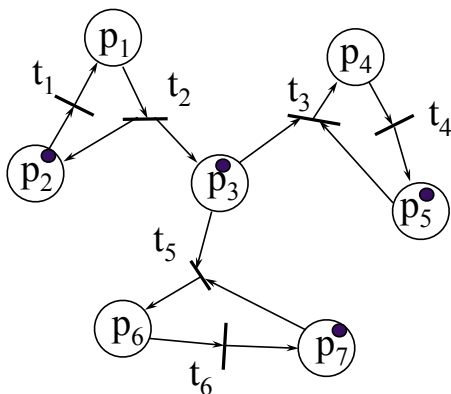
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- ◆ Example: assume the following initial marking
 - Only one transition is enabled, i.e. t_2



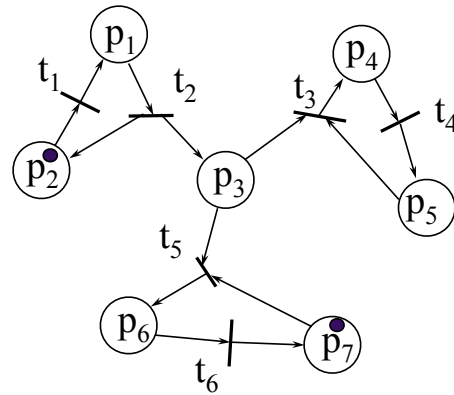
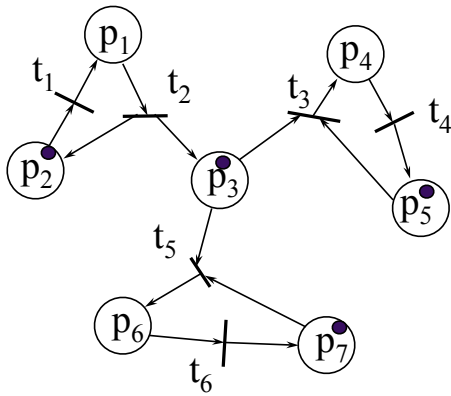
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- Now several transitions are enabled, i.e. t_1 , t_3 and t_5
- if t_1 fires first



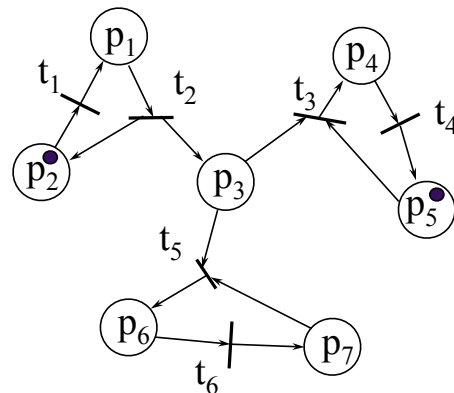
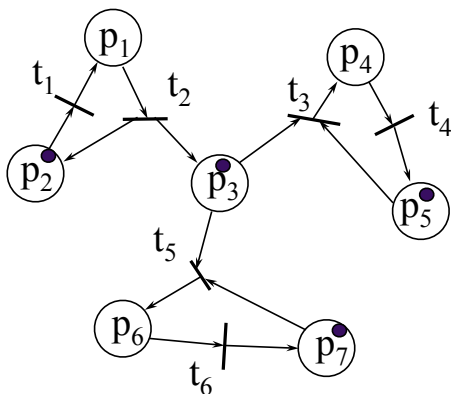
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- if t_3 fires first



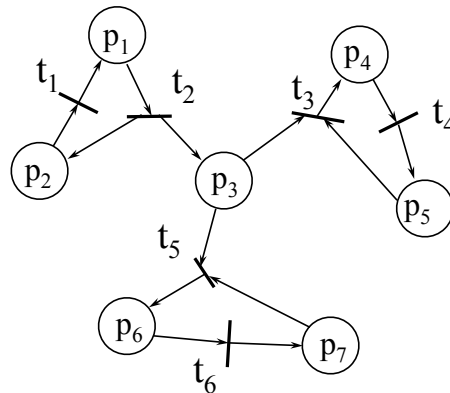
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- if t_5 fires first
- t_3 and t_5 are said to be in conflict



Petri Nets

- what could this Petri net represent?



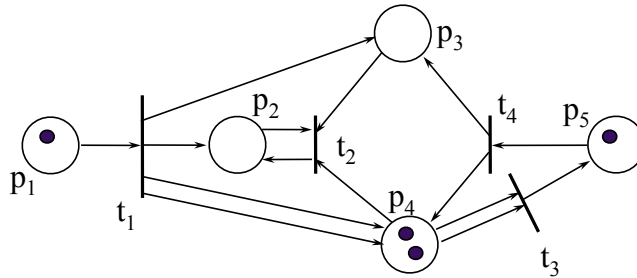
Petri Nets

- ◆ Marking: Number and placement of tokens
 - let $m_i = \#$ of tokens in place p_i
 - then marking
$$M = \{m_1, m_2, \dots, m_n\}$$
 - marking -- system state
 - Advantage: economy of model
 - » e.g. assume net with 6 places
 - we limit each place to maximal 1 token
 - then there are 2^6 possible markings
 - \Rightarrow 64 states
 - thus Petri Nets are a lot smaller than state diagrams, i.e. Markov chains

Petri Nets

◆ Firing rules

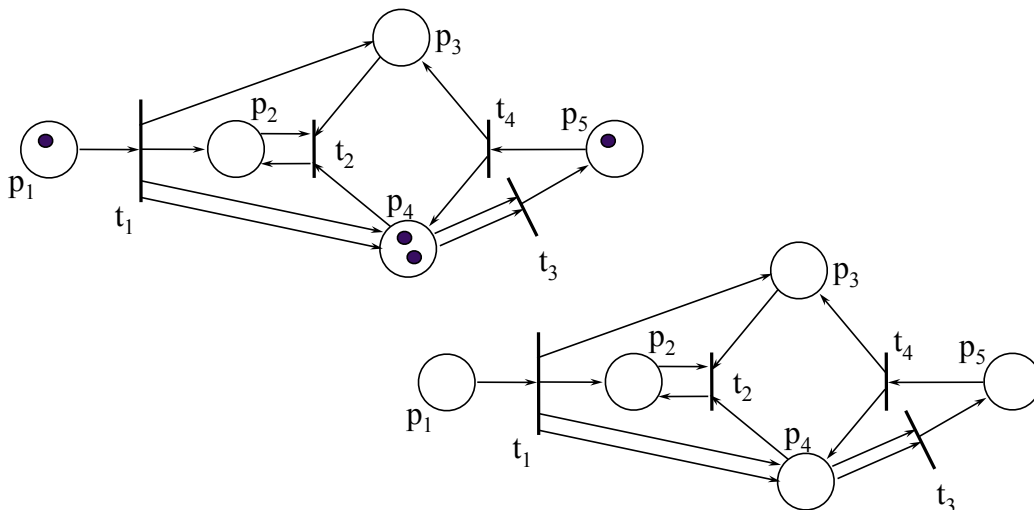
- transition 1,3 and 4 are enabled



Petri Nets

◆ Firing rules

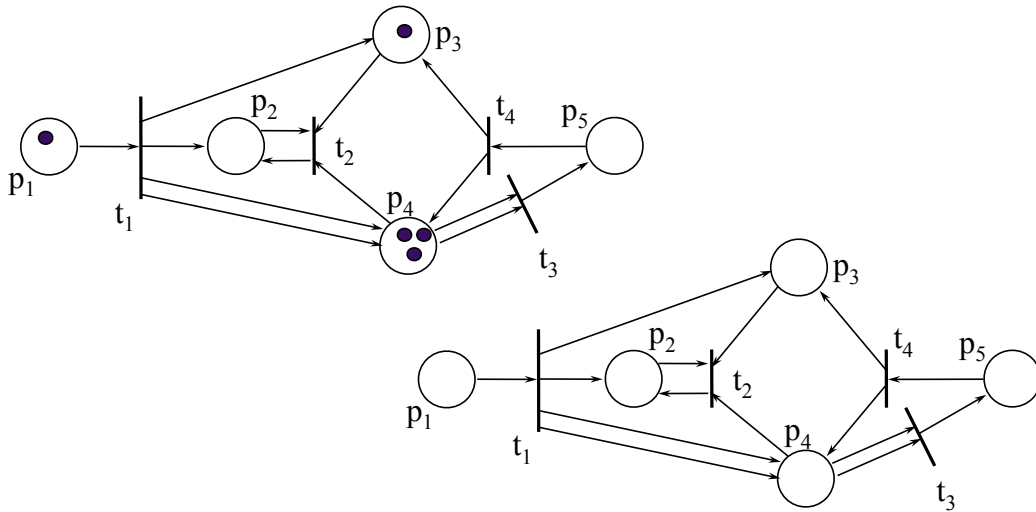
- transition 4 fires



Petri Nets

◆ Firing rules

- transition 1 fires



Petri Nets

◆ Firing rules

- transition 3 fires

