Petri Nets

- Part of this discussion is based on the paper
  - *Petri Nets: Properties, Analysis and Applications*

- **Petri Nets**
  - graphical and mathematical modeling tool
  - tool for describing systems characterized as being:
    - concurrent, asynchronous, distributed, parallel, nondeterministic and/or stochastic

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Petri Nets

- **History**
  - **1962**: Carl Adam Petri’s submitted his dissertation at the Uni. Darmstadt, Germany
  - **1970**: early development was published by A.W. Host and in the records of the 1970 Project MAC Conference on Concurrent Systems and Parallel Computation
  - **1970-75**: Computation Structure Group and MIT was most active
  - **1975**: conference on Petri Nets and Related Methods at MIT
  - **1979**: 135 researchers assembled in Hamburg, Germany, for 2-week advanced course on General Net Theory of Processes and Systems
  - **1980**: first European Workshop on Applications and Theory of Petri Nets, Strasbourg, France.
  - check out Murata’s paper for the extensive literature discussion
Petri Nets

- General:
  - directed, weighted, bipartite graph
  - two kinds of notes (Places P, Transitions T)
  - arcs from P to T or from T to P
  - arcs have integer weights
  - non-negative Place weights are called tokens

A Petri Net is a 5-touple PN={P,T,A,W,M0}

- Place Set P = \{p_1, p_2, ..., p_m\}
  - finite set of places
  - condition = place
  - one condition or set of atomic conditions
  - symbol

- Transition Set T = \{t_1, t_2, ..., t_n\}
  - finite set of transitions
  - action = transition
  - one action or set of atomic transitions
  - symbol
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- **Arc Set** $A \subseteq (P \times T) \cup (T \times P)$
  - set of directed arcs
  - edge of graph = arc
  - symbol $\rightarrow$

- **Weight Function** $W = A \rightarrow \{1, 2, 3, \ldots\}$
  - weights are associated with arcs

- **Initial Marking** $M_0 = P \rightarrow \{0, 1, 2, \ldots\}$
  - the initial assignment of tokens to places

**example**

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**Petri Nets**

- **Dynamic Behavior**
  - during simulation of a petri net the state of the net may change
  - change of state:
    » transitions can be enabled
    » enabled transitions may fire
    » firing transition changes the marking of the net
    » the marking is the “snap-shot” of all the tokens

**Petri Nets**

- **Firing rules**
  - A transition $T$ is said to be *enabled* if each input place $P$ is marked with at least $W(P,T)$ tokens
    » $W(P,T)$ is the weight of the arc from $P$ to $T$
  - An enabled transition may or may not fire (depending on whether or not the event actually takes place).
  - A firing of an enabled transition $T$ removes $W(P,T)$ tokens from each input place $P$ of $T$, and adds $W(T,P)$ tokens to each output place $P$ of $T$
    » $W(T,P)$ is the weight of the arc from $T$ to $P$
  - Common misconception: When a transition fires, it does not move tokens
    » i.e. the number of tokens in the system is not necessarily constant
**Petri Nets**

- Example: assume the following initial marking
  - Only one transition is enabled, i.e. $t_2$

- Now several transitions are enabled, i.e. $t_1$, $t_3$, and $t_5$
  - if $t_1$ fires first
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- if $t_3$ fires first

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- if $t_5$ fires first
  - $t_3$ and $t_5$ are said to be in conflict
**Petri Nets**

- what could this Petri net represent?

![Petri Net Diagram]

**Marking:** Number and placement of tokens
- let $m_i = \#$ of tokens in place $p_i$
- then marking
  \[ M = \{m_1, m_2, \ldots, m_n\} \]
- marking -- system state
- Advantage: economy of model
  - e.g. assume net with 6 places
    - we limit each place to maximal 1 token
    - then there are $2^6$ possible markings
    - => 64 states
    - thus Petri Nets are a lot smaller than state diagrams, i.e. Markov chains
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- Firing rules
  - transition 1, 3 and 4 are enabled

![Petri Net Diagram with Firing Rules](image1)

- Firing rules
  - transition 4 fires

![Petri Net Diagram with Firing Rules](image2)
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- Firing rules
  - transition 1 fires

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- Firing rules
  - transition 3 fires