Petri Nets

- Part of this discussion is based on the paper
  - *Petri Nets: Properties, Analysis and Applications*
  - by Tadoa Murata, Proc. IEEE, Vol. 77, No. 4, April 1989.
- Petri Nets
  - graphical and mathematical modeling tool
  - tool for describing systems characterized as being:
    - concurrent, asynchronous, distributed, parallel, nondeterministic and/or stochastic

Petri Nets

- History
  - **1962:** Carl Adam Petri's submitted his dissertation at the Uni. Darmstadt, Germany
  - **1970:** early development was published by A.W. Host and in the records of the 1970 Project MAC Conference on Concurrent Systems and Parallel Computation
  - **1970-75:** Computation Structure Group and MIT was most active
  - **1975:** conference on Petri Nets and Related Methods at MIT
  - **1979:** 135 researchers assembled in Hamburg, Germany, for 2-week advanced course on General Net Theory of Processes and Systems
  - **1980:** first European Workshop on Applications and Theory of Petri Nets, Strasbourg, France.
  - check out Murata's paper for the extensive literature discussion

Petri Nets

- General:
  - directed, weighted, bipartite graph
  - two kinds of notes (Places P, Transitions T)
  - arcs from P to T or from T to P
  - arcs have integer weights
  - non-negative Place weights are called tokens

Petri Nets

- A Petri Net is a 5-touple PN={P,T,A,W,M0}
- Place Set P = $\{p_1, p_2, ..., p_m\}$
  - finite set of places
  - condition = place
  - one condition or set of atomic conditions
  - symbol ○
- Transition Set T = $\{t_1, t_2, ..., t_n\}$
  - finite set of transitions
  - action = transition
  - one action or set of atomic transitions
  - symbol ―

Petri Nets

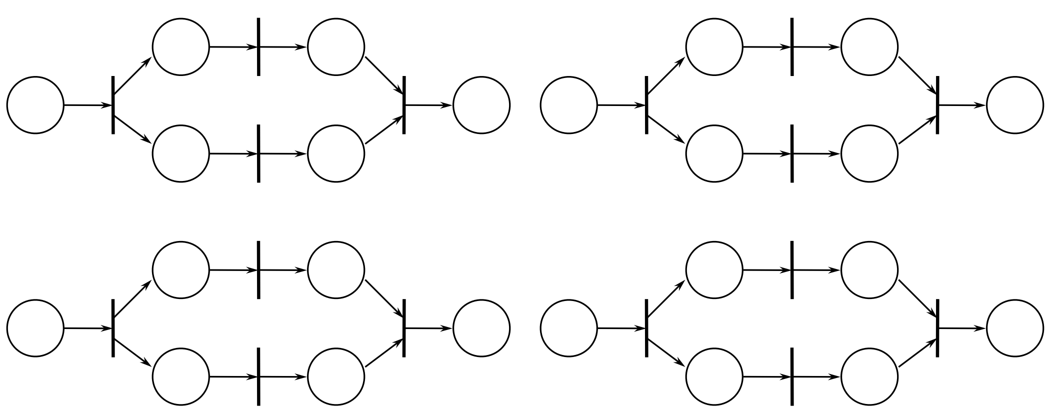
- Arc Set $A \subseteq (P \times T) \cup (T \times P)$
  - set of directed arcs
  - edge of graph = arc
  - symbol $\longrightarrow$

- Weight Function $W = A \rightarrow \{1,2,3,\ldots\}$
  - weights are associated with arcs

- Initial Marking $M_0 = P \rightarrow \{0,1,2,\ldots\}$
  - the initial assignment of tokens to places

Petri Nets

- example



Petri Nets

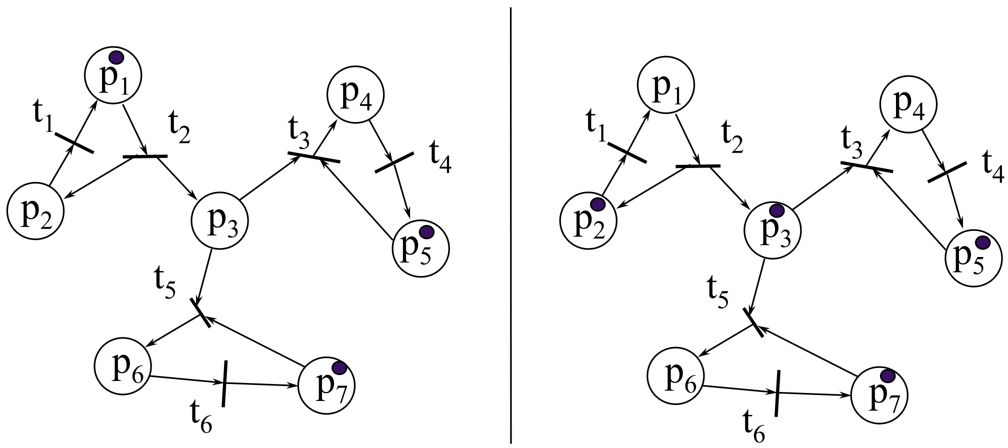
- Dynamic Behavior
  - during simulation of a petri net the state of the net may change
  - change of state:
    - transitions can be enabled
    - enabled transitions may fire
    - firing transition changes the marking of the net
    - the marking is the "snap-shot" of all the tokens

Petri Nets

- Firing rules
  - A transition *T* is said to be *enabled* if each input place *P* is marked with at least *W(P,T)* tokens
    - *W(P,T)* is the weight of the arc from *P* to *T*
  - An enabled transition may or may not fire (depending on whether or not the event actually takes place).
  - A *firing* of an enabled transition *T* removes *W(P,T)* tokens from each input place *P* of *T*, and adds *W(T,P)* tokens to each output place *P* of *T*
    - *W(T,P)* is the weight of the arc from *T* to *P*
  - Common misconception: When a transition fires, it does **not** *move* tokens
    - i.e. the number of tokens in the system is not necessarily constant

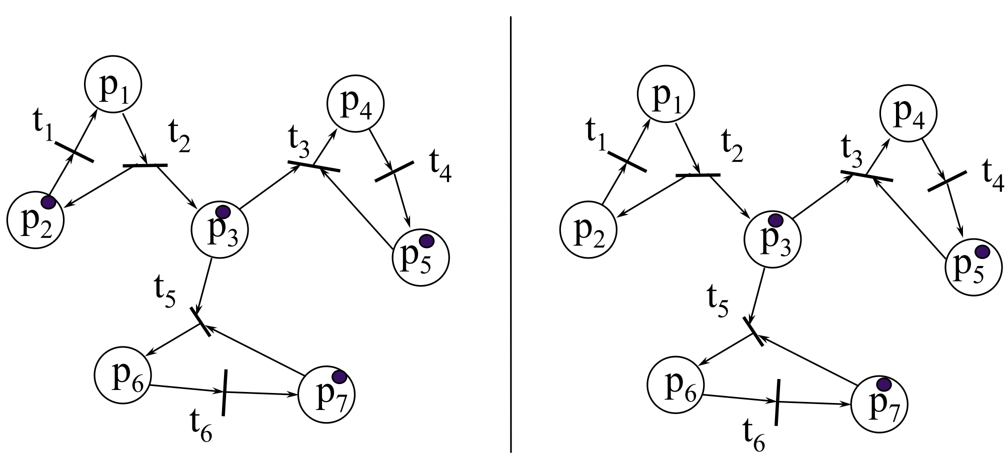
Petri Nets

- Example: assume the following initial marking
  - Only one transition is enabled, i.e. t_2



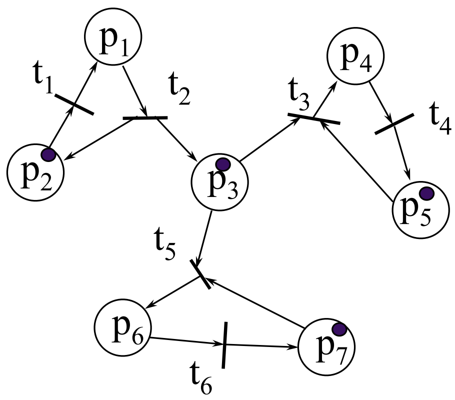
Petri Nets

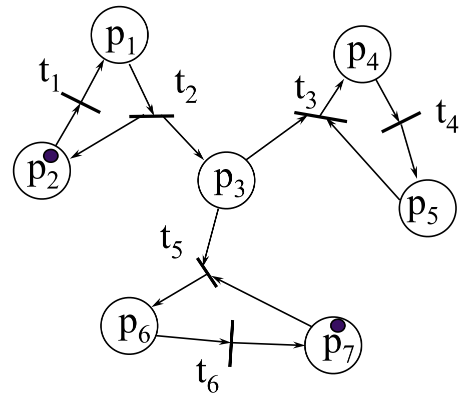
- Now several transitions are enabled, i.e. t_1 t_3 and t_5
- if t_1 fires first



Petri Nets

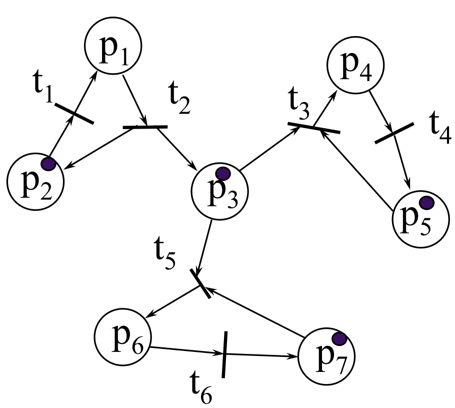
– if t_3 fires first

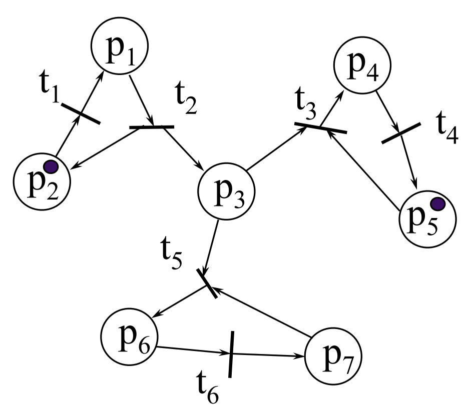




Petri Nets

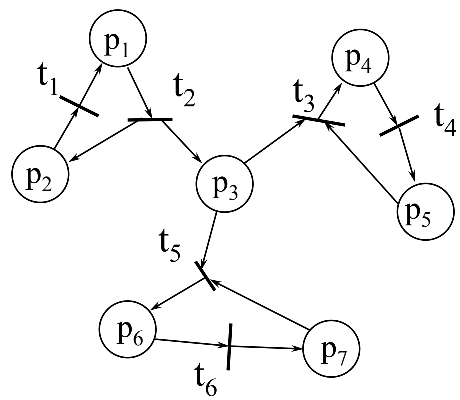
– if t_5 fires first
– t_3 and t_5 are said to be in conflict





Petri Nets

- what could this Petri net represent?



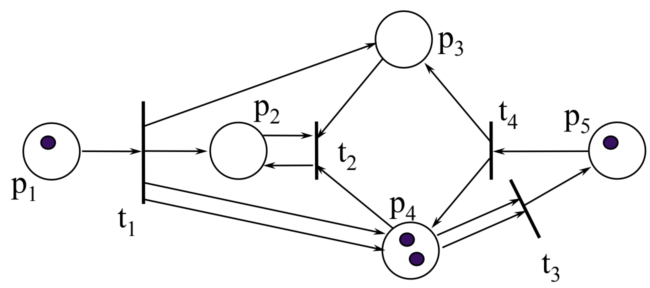
Petri Nets

- Marking: Number and placement of tokens
  - let m_i = # of tokens in place p_i
  - then marking

    $M = \{m_1, m_2 , ..., m_n\}$
  - marking -- system state
  - Advantage: economy of model
    - e.g. assume net with 6 places
      - we limit each place to maximal 1 token
      - then there are 2^6 possible markings
      - => 64 states
      - thus Petri Nets are a lot smaller than state diagrams, i.e. Markov chains

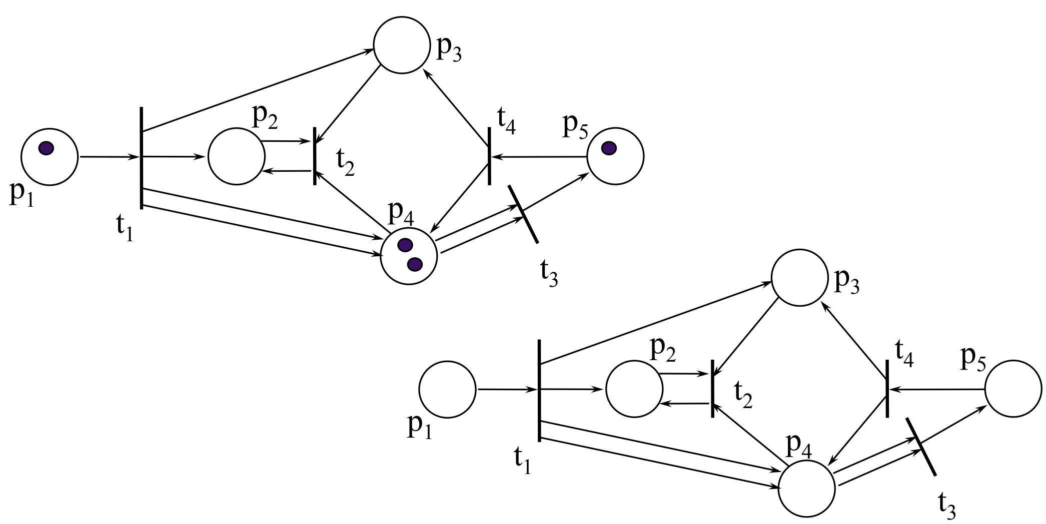
Petri Nets

- Firing rules
  - transition 1,3 and 4 are enabled



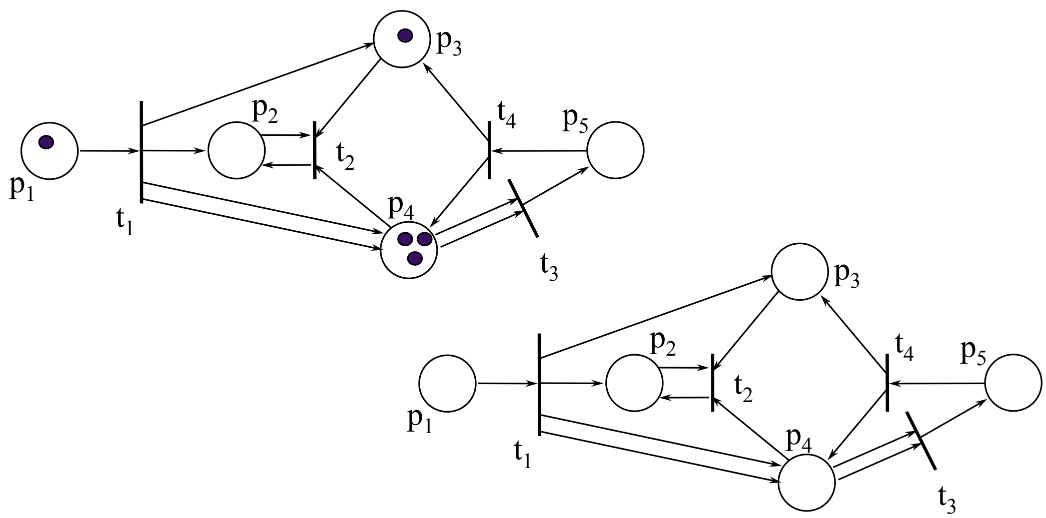
Petri Nets

- Firing rules
  - transition 4 fires



Petri Nets

- Firing rules
  - transition 1 fires



Petri Nets

- Firing rules
  - transition 3 fires