PRA & PSA

- Probability Risk Assessment
  - PRA
- Probability Safety Assessment
  - PSA
- Fault Tree Analysis
  - FTA
- Event Tree Analysis
  - ETA

PRA & PSA

- Probability Risk/Safety Assessment
  - general term for risk assessments that use probability models to represent the likelihood of different risk levels
  - reliability assessment methods used to analyze systems which are considered critical
  - PSA normally deals with issues of safety
  - PRA may deal with non-safety issues

Definitions

- Variability
  - true heterogeneity or diversity
  - example: drinking water
    - » for different people the risk from consuming the water may vary
    - » could be caused by different body weight, exposure duration & frequency

Definitions

- Uncertainty
  - caused by lack of knowledge
  - example: drinking water
    - » risk assessor is certain that different people consume different amounts of water
    - » BUT may be uncertain about how much variability there is

Definitions

- Random Variable X
  - a function that assigns a real number $X(s)$ to each sample point s in sample space S
  - e.g. coin toss, number of heads in a sequence of 3 tosses
  -
  -

s	hhh	hht	hth	htt	thh	tht	tth	ttt
$X(s)$	3	2	2	1	2	1	1	0

  - X is a random variable taking on values in the set

$$S_X = \{0,1,2,3\}$$

Definitions

- Cumulative Distribution Function (cdf)
  - The cdf of a random variable X is defined as the probability of the event $\{X \le x\}$

$$F_X(x) = P(X \le x) \text{ for } -\infty < x < +\infty$$

$$F_X(x) = \text{prob. of event } \{s: X(s) \le x\}$$

$$F_X(x) = \text{is a probability, i.e. } 0 \le F_X(x) \le 1$$

$$F_X(x) \text{ is monotonically non-decreasing,}$$
$$\text{i.e. if } x_1 \le x_2 \text{ then } F_X(x_1) \le F_X(x_2)$$

$$\lim_{x \to \infty} F_X(x) = 1 \qquad \lim_{x \to -\infty} F_X(x) = 0$$

Definitions

- Probability Density Function (pdf)
  - The pdf of a random variable is the derivation of $F_X(x)$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

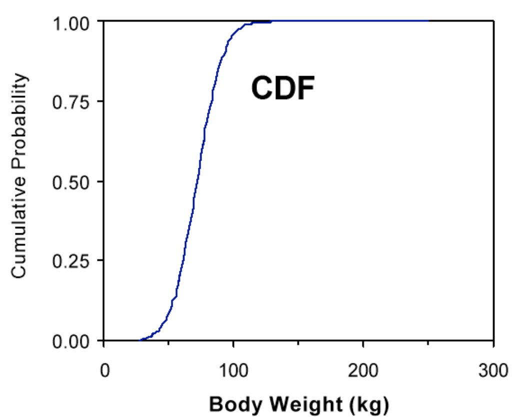
  - Since $F_X(x)$ is a non-decreasing function,

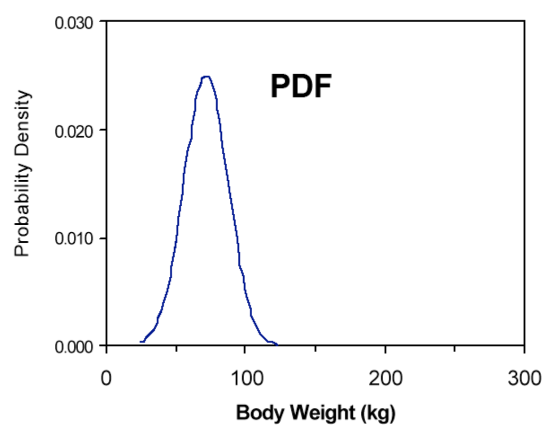
$$f_X(x) \geq 0$$

  - The pdf represents the “density” of probability at point x

Definitions

- cdf vs. pdf
  - adult body weight (males and females combined)
  - Arithmetic mean 71.7kg, std = 15.9kg
  - Source: Finley et.al. 1994





Definitions

- Expectation of a random variable
  - in order to completely describe the behavior of a random variable, an entire function, namely the cdf or pdf, must be given
  - however, sometime we are just interested in parameters that summarize information

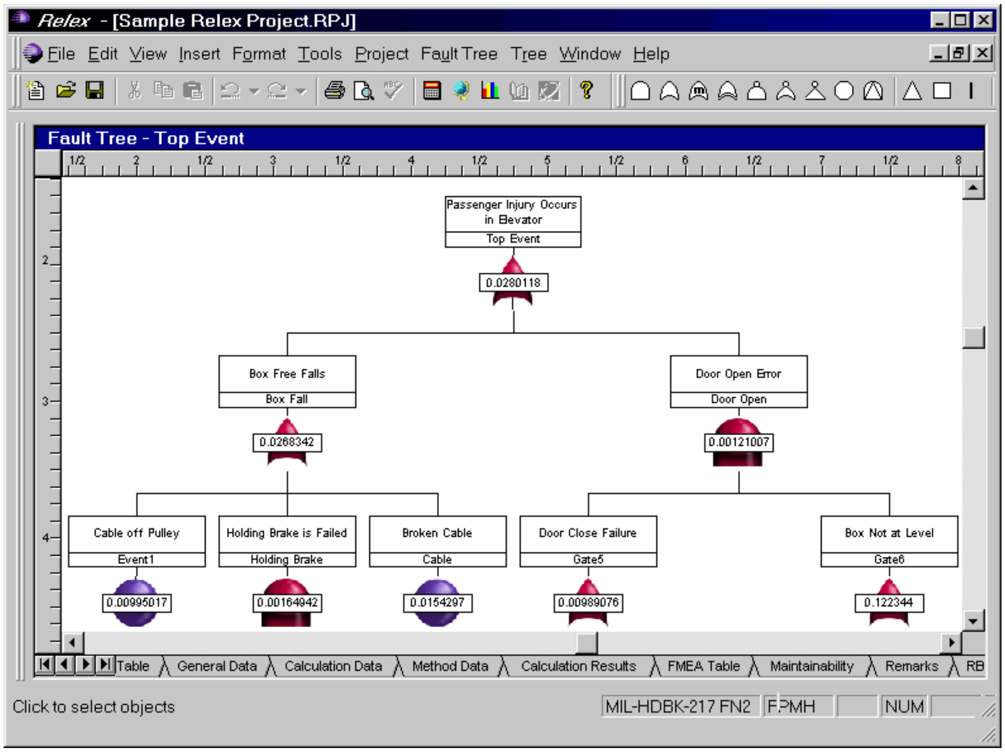
$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$

i.e. mean time to failure = expected lifetime of the system

PRA & PSA

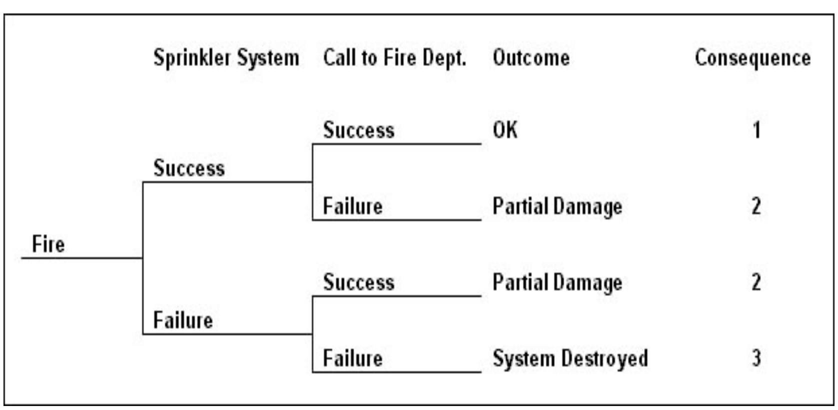
- Fault Tree Analysis
  - most widely used method in system reliability analysis
  - this is a top down approach
  - typical components are AND and OR

– example: (source Relax Software Corp.)



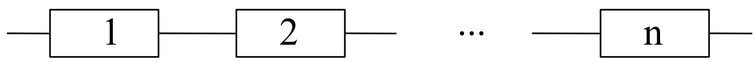
PRA & PSA

- Event Tree Analysis
  - visual representation of all events which can occur in a system
  - example: (source Relax Software Corp.)



Reliability of Series System

- Any one component failure causes system failure
- Reliability Block Diagram (RBD)



$$R(t)_{\text{series}} = \prod_{i=1}^{n} R_i(t)$$

$$= \prod_{i=1}^{n} e^{-\lambda_i t}$$

$$= e^{-(\sum_{i=1}^{n} \lambda_i) t}$$

Reliability of Series System

thus $$\lambda_{\text{series}} = \sum_{i=1}^{n} \lambda_{\text{i}}$$

Mean time to failure of series system:

$$MTTF_{\text{series}} = \frac{1}{\sum_{i=1}^{n} \lambda_{\text{i}}}$$

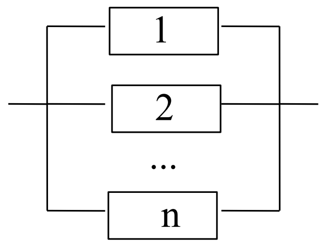
Thus the MTTF of the series system is much smaller than the MTTF of its components

if $X_i \equiv$ lifetime of component i then
$0 \le E[X] \le \min\{E[X_i]\}$

system is weaker than weakest component

Reliability of Parallel System

- All components must fail to cause system failure
- Reliability Block Diagram (RBD)



  - assume mutual independence

X is lifetime of the system

$$X = \max\{X_1, X_2, \ldots, X_n\}$$ n components

$$R(t)_{\text{parallel}} = 1 - \prod_{i=1}^{n} Q_i(t)$$

$$= 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

$$\geq 1 - (1 - R_i(t))$$

Assuming all components have exponential distribution with parameter λ

$$R(t) = 1 - (1 - e^{-\lambda t})^n$$

$$E(X) = \int_0^{\infty} [1 - (1 - e^{-\lambda t})^n] dt$$

$$= \quad \dots$$

$$= \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i}$$

$$\approx \frac{\ln(n)}{\lambda}$$

from previous page

$$Q(t)_{\text{parallel}} = \prod_{i=1}^{n} Q_i(t)$$

Product law of unreliability

Stand-by Redundancy

- When primary component fails, standby component is started up.
- Stand-by spares are cold spares => unpowered
- Switching equipment assumed failure free

Let X_i denote the lifetime of the i-th component from the time it is put into operation until its failure.

System lifetime:

$$X_{sys} = \sum_{i=1}^{n} X_i$$

Stand-by Redundancy

- MTTF $E(X) = \frac{n}{\lambda}$
  - gain is linear as a function of the number of components, unlike the case of parallel redundancy
  - added complexity of detection and switching mechanism

M-of-N System

Starting with N components, we need any M components operable for the system to be operable.

Example: TMR

$$R_{\mathrm{TMR}}(t) = R_1(t)R_2(t)R_3(t) + R_1(t)R_2(t)(1 - R_3(t)) + R_1(t)(1 - R_2(t))R_3(t) + (1 - R_1(t))R_2(t)R_3(t)$$

Where $R_i(t)$ is the reliability of the i-th component

if $R_i(t) = R_1(t) = R_2(t) = R_3(t) = R(t)$ then

$$\begin{aligned} R_{\mathrm{TMR}}(t) &= R^3(t) + 3R^2(t)(1 - R(t)) \\ &= R^3(t) + 3R^2(t) - 3R^3(t) \\ &= 3R^2(t) - 2R^3(t) \end{aligned}$$

M-of-N System

The probability that exactly *j* components are not operating is

$$\binom{N}{j} Q^{j}(t) R^{N-j}(t) \qquad \text{with} \quad \binom{N}{j} = \frac{N!}{j!(N-j)!}$$

then

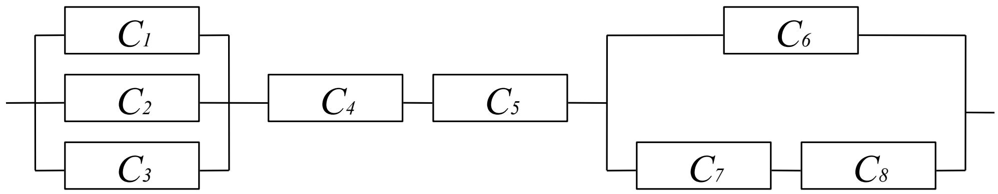
$$R_{MofN}(t) = \sum_{i=0}^{N-M} \binom{N}{i} Q^{i}(t) R^{N-i}(t)$$

Reliability Block Diagram

- Series Parallel Graph
  - a graph that is recursively composed of series and parallel structures.
  - therefore it can be “collapsed” by applying series and/or parallel reduction
  - Let C_i denote the condition that component *i* is operable
    - » 1 = up, 0 = down
  - Let *S* denote the condition that the system is operable
    - » 1 = up, 0 = down
  - *S* is a logic function of *C*’s

Reliability Block Diagram

- Example:



$$S = (C_1 + C_2 + C_3)(C_4 C_5)(C_6 + C_7 C_8)$$

$+$ => parallel (1 of N)

$.$ => series (N of N)

K of N system

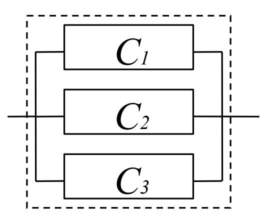
- Example 2-of-3 system

$$S = (C_1 C_2 + C_1 C_3 + C_2 C_3)$$

may abbreviate

$$S = \tfrac{2}{3}(C_1 C_2 C_3)$$

draw as parallel



2-of-3

Fault Trees

- Fault Trees
  - dual of Reliability Block Diagram
  - logic failure diagram
  - think in terms of logic where
    - 0 = operating,   1 = failed
- AND Gate
  - all inputs must fail for the gate to fail
- OR Gate
  - any input failure causes the gate to fail
- k-of-n Gate
  - k or more input failures cause gate to fail