Before dealing with the issue of usage models we need to cover/review the basics of Markov chains.
- some of you have seen this material in other classes
- we will only discuss the very basics
- for details about Markov models, please refer to the general literature
Usage Model & Markov Process

- Markov Analysis of Software Specifications
  - this can be used as a tool in determining “what is normal behavior”
  - anything “not normal” must deviate from the normal behavior in some way.

- A stochastic process is a function whose values are random variables

- The classification of a random process depends on different quantities
  - state space
  - index (time) parameter
  - statistical dependencies among the random variables X(t) for different values of the index parameter t.
Markov Process

- **State Space**
  - the set of possible values (states) that $X(t)$ might take on.
  - if there are finite states $\Rightarrow$ discrete-state process or chain
  - if there is a continuous interval $\Rightarrow$ continuous process

- **Index (Time) Parameter**
  - if the times at which changes may take place are finite or countable, then we say we have a discrete-(time) parameter process.
  - if the changes may occur anywhere within a finite or infinite interval on the time axis, then we say we have a continuous-parameter process.
Markov Process

- In 1907 A.A. Markov published a paper in which he defined and investigated the properties of what are now known as Markov processes.
- A Markov process with a discrete state space is referred to as a Markov Chain.
- A set of random variables forms a Markov chain if the probability that the next state is $S_{(n+1)}$ depends only on the current state $S_{(n)}$, and not on any previous states.
Markov Process

- States must be
  - mutually exclusive
  - collectively exhaustive

- Let $P_i(t) =$ Probability of being in state $S_i$ at time $t$.

$$\sum_i P_i(t) = 1$$

- Markov Properties
  - future state prob. depends only on current state
    » independent of time in state
    » path to state
Markov Process

- Let $P(\text{transition out of state } i \text{ in } \Delta t) = \sum_{j \neq i} \lambda_{ij} \Delta t$
- Mean time to transition (exponential holding times)
  \[
  \frac{1}{\sum_{j \neq i} \lambda_{ij}}
  \]
- If $\lambda$’s are not functions of time, i.e. if $\lambda_i \neq f(t)$
  - homogeneous Markov Chain
Markov Process

- **Accessibility**
  - State $S_i$ is accessible from state $S_j$ if there is a sequence of transitions from $S_j$ to $S_i$.

- **Recurrent State**
  - State $S_i$ is called recurrent, if $S_i$ can be returned to from any state which is accessible from $S_i$ (in one step).

- **Non Recurrent**
  - If there exists at least one neighbor with no return path.
Markov Process

- sample chain

Which states are recurrent or non-recurrent?
Markov Process

- Classes of States
  - set of states (recurrent) s.t. any state in the class is reachable from any other state in the class.
  - note: 2 classes must be disjoint, since a common state would imply that states from both classes are accessible to each other.

- Absorbing State
  - a state $S_i$ is absorbing iff

$$\sum_{j \neq i} \lambda_{ij} \Delta t = 0$$
Markov Process

- Irreducible Markov Chain
  - a Markov chain is called irreducible, if the entire system is one class
    => there is no absorbing state
    => there is no absorbing subgraph,
    i.e. there is no absorbing subset of states

- What if the chain is not irreducible?
  - what are the implications?
Markov Process

- Steady State Solution
  - as time goes toward infinity

- Transient Solution
  - when is this of importance?