Example Fibonacci

Fibonacci numbers:

0, 1, 1, 3, 5, 8, 13, 21, 34, 55, ...

Recurrence:

$F(n) = F(n\text{-}1) + F(n\text{-}2)$ for $n > 1$

Initial conditions:

$F(0) = 0$, $F(1) = 1$

Shall we use backward substitution?

Example Fibonacci

Recurrence:

$F(n) = F(n\text{-}1) + F(n\text{-}2)$ for $n > 1$ with $F(0) = 0,\quad F(1) = 1$

is a 2nd order linear homogeneous recurrence with constant coefficients:

$$aX(n) + bX(n\text{-}1) + cX(n\text{-}2) = 0$$

Solving $aX(n) + bX(n-1) + cX(n-2) = 0$

1) Set up the characteristic equation (quadratic)

$$ar^2 + br + c = 0$$

2) Solve to obtain roots r_1 and r_2

3) General solution to the recurrence

if r_1 and r_2 are two distinct real roots: $X(n) = \alpha r_1^n + \beta r_2^n$

if $r_1 = r_2 = r$ are two equal real roots: $X(n) = \alpha r^n + \beta n r^n$

4) Particular solution can be found by using initial conditions

Linear Recurrence Relations