Example Fibonacci

Fibonacci numbers:
0, 1, 1, 3, 5, 8, 13, 21, 34, 55, ...

Recurrence:
\[ F(n) = F(n-1) + F(n-2) \quad \text{for } n > 1 \]

Initial conditions:
\[ F(0) = 0, \quad F(1) = 1 \]

Shall we use backward substitution?
Example Fibonacci

Recurrence:

\[ F(n) = F(n-1) + F(n-2) \text{ for } n > 1 \text{ with } F(0) = 0, \quad F(1) = 1 \]

is a 2\textsuperscript{nd} order linear homogeneous recurrence with constant coefficients:

\[ aX(n) + bX(n-1) + cX(n-2) = 0 \]
Solving \( aX(n) + bX(n-1) + cX(n-2) = 0 \)

1) Set up the characteristic equation (quadratic)
\[
ar^2 + br + c = 0
\]

2) Solve to obtain roots \( r_1 \) and \( r_2 \)

3) General solution to the recurrence
   - if \( r_1 \) and \( r_2 \) are two distinct real roots: \( X(n) = \alpha r_1^n + \beta r_2^n \)
   - if \( r_1 = r_2 = r \) are two equal real roots: \( X(n) = \alpha r^n + \beta nr^n \)

4) Particular solution can be found by using initial conditions
Linear Recurrence Relations