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4a) Answer in text

4B) $L = \{a^n b^l a^k : k \neq n+l\}$

Assume N as in the P.L.
(Example 4.13 is a good place to look for hints)

Choose $\omega = a^{N!} b a^{(N+1)!+1}$

w/ $\omega = xyz$ & $|xy| \leq N$ y must be some number of a's from the 1st part of the string, say p is p a's
i.e. $y = a^p$

Now to contradict the P.L. and prove L is not regular we need to show that for any $p \leq N$ the following:

$N! + (i-1)p + 1 = (N+1)! + 1$ is true for some i

$+1$ for the b ; $+1$ the extra a to balance the b

Solve the equation

$$N! + (i-1)p + 1 = (N+1)! + 1$$

$$(i-1)p = (N+1)! - N! \quad \text{'pull out' an } N!$$

$$(i-1)p = N!(N+1-1) = N!N$$

$$i = \frac{N!N}{p} + 1$$ which is the i to make $\omega_i \notin L$ invalidating the P.L. and proving L is not regular

Pg. 122

14) Prove or disprove: if L_1 and L_2 are non-regular $L_1 \cup L_2$ is non-regular.

Although this seems reasonable, two non-regular languages combine into a non-regular language, it is <u>not</u> always true (although for some languages it is true).

Proof:

Consider the language: $L_1 = \{a^n b^n \; n \geq 0\}$

L_1 is non-regular

$L_2 = \bar{L}_1$ is also non-regular because regular languages are closed under complement (if L_2 were regular $L_2 = \bar{L}_1 = L_1$ would have to be regular, but its not, so L_2 isn't).

Now consider:

$L_2 \cup L_1 = \bar{L}_1 \cup L_1 =$ all strings which is regular.

So the union of two non-regular languages can be regular.

15 a, b) done in the text

24. We know $L_1 \cup L_2$ is regular
and L_1 is regular

Is L_2 necessarily regular? No, although it might be.

Proof:

Let L_1 be all strings. E.g. over the alphabet $\{a, b\}$ $L_1 = L((a+b)^*) = L(M_1)$ where M_1 is

Clearly L_1 is regular (I just wrote a regular expression and a DFA for it).

$L_1 \cup L_2 = L_1$ also regular

So, whether L_2 is regular or not L_1 and $L_1 \cup L_2$ are.

26 $L = \{a^n b^m : n \geq 100 \; m \leq 50\}$

a) No. You can not use the P.L. to prove L is regular because the P.L. can only be used to prove that languages are not regular.

b) No. Because L is regular.