Assignment #9 Pg. 212 1, 7c, 8b, 8c, 10 Pg 220: 7, 12, 17

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1) $L = \{w \in \{a,b,c\}^* : n_a(w) = n_b(w) \leq n_c(w)\}$

Let m be as in the P.L.

Pick $w = a^m b^m c^m$

$w = uvxyz$ w/ $|vxy| \leq m$ $|vy| \geq 1$

because $|vxy| \leq m$ vy can be:

Just a's, for any $i > 1$ $n_a(w_i) \neq n_b(w_i)$ so $w_i \notin L$

Just b's " $n_a(w_i) \neq n_b(w_i)$ so $w_i \notin L$

Just c's pick $i = 0$ then w_i has too few c's $n_c(w_i) < n_b(w_i)$ so $w_i \notin L$

a's & b's for any $i > 1$ $n_b(w_i) > n_c(w_i)$ so $w_i \notin L$

b's & c's for any $i \neq 1$ $n_b(w_i) \neq n_a(w_i)$ so $w_i \notin L$

So for any valid vxy there is an i that forces w_i out of the language. This violates the P.L. and proves that L is not context free.

7c) $L = \{a^n b^j c^k : k = nj\}$

Let m be as in the P.L.

Pick $w = a^m b^m c^{m^2} \in L$

$w = uvxyz$ w/ $|vxy| \leq m$ and $|vy| \geq 1$

For the cases vy equals just a's, just b's, just c's or a's & b's any value of i not equal to 1 will mean $k \neq jn$ and thus $w_i \notin L$

The only tricky case is vy equals some b's and some c's

Say vy is x b's & y c's

Then the resulting equation is (if w_i is in L)

$$\underbrace{m}_{a's} \cdot \underbrace{(m + i(x-1))}_{b's} = \underbrace{m^2 + i(y-1)}_{c's} \quad \text{for } w_i$$

$$\Downarrow$$

$$\cancel{m^2} + mi(x-1) = \cancel{m^2} + i(y-1)$$

pick $i = m$ to get

$$m^2(x-1) = m(y-1)$$

$$m(x-1) = (y-1) \quad \text{but } (y-1) < m \text{ because } y \leq m$$

So this can't be true.

Thus $w_i = w_m \notin L$ and L is not context free.

8b) In text

8c) $L = \{a^n b^j a^j b^n, n \geq 0, j \geq 0\}$

Because of the ordering this "Looks" context free.

The grammar

$$S \rightarrow aSb \mid A$$
$$A \rightarrow bAa \mid \lambda$$ proves it.

10) in text

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7) Show that if L_1 is context free and L_2 is regular then $L_1 - L_2$ is context free.

Difference is "related to" intersection and by theorem 8.5 and Context Free language are closed under regular intersection, so this suggests the following proof:

$L_1 - L_2 = L_1 \cap \overline{L_2}$ Regular languages are closed under complement so $\overline{L_2}$ is regular.

Context Free Languages are closed under regular intersection so $L_1 \cap \overline{L_2}$ is contect free.

Thus $L_1 - L_2$ is contect free (for L_1 context free & L_2 regular)

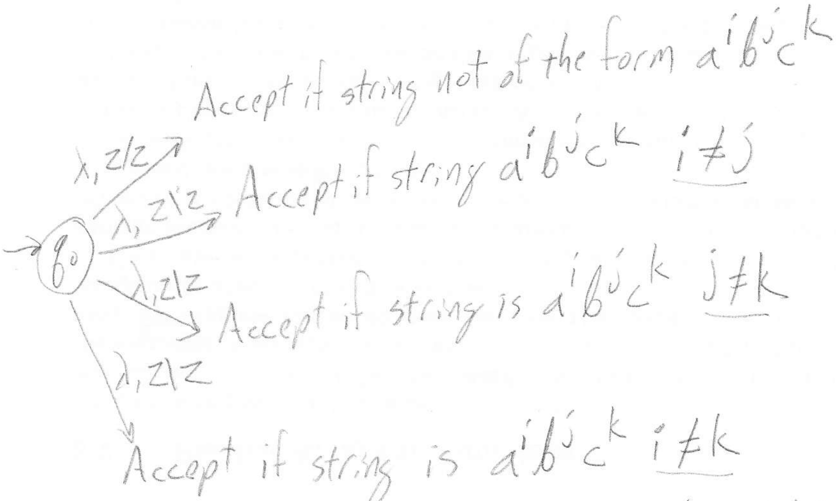
12) (Its often easier to start w/ non-context free languges and find one whose complement is regular.)

Try $L = \{a^n b^n c^n ; n \geq 0\}$ which is not C.F.

The complement is all strings either not of the form $a^i b^j c^k$ or of the form $a^i b^j c^k$ where $i \neq j$ or $j \neq k$ or $i \neq k$

This can be accepted by a NPDA using non-determinism.

Here's a sketch of the machine



Works because only one condition has to be checked.

Note that no branch accepts $a^n b^n c^n$ so this accepts $\overline{L}$.

17) $L = \{a^n b^n : n \geq 0$ n is not a multiple of $5\}$

(Discussed in class)

One option is to use closure properties;

$L' = \{a^i : i$ is not a multiple of $5\}$ is <u>regular.</u>

Thus, by closure of regular languages under concatenation

$L'' = \{a^i b^j : i \& j$ are not multiples of $5\}$ is regular

$L = \underbrace{\{a^n b^n ; n \geq 0\}}_{\text{Context Free}} - L''$ by regular difference (see problem 7) L is context free

Alternatively the C.F. grammar;

$S \to A \mid \lambda$

$A \to aaaaaAbbbbb \mid B$

$B \to ab \mid aabb \mid aaabbb \mid aaaabbbb$

proves it.

(Note the trick, all strings are multiples of 5 until the last step, which forces them not to be. This could also be done in reverse start w/ a non-multiple of 5 then add only multiples.)