Assignment #6/7 key (4a, 4b, 4c, 5a, 14, 15a, 15b, 24, 26, Page 133: 2, 3, 7a)
Page 122.

4a) Answer in text

4B) $L = \{a^n b^l a^k ; k \neq n+l\}$

Assume N as in the P.L.
(Example 4.13 is a good place to look for hints)

Choose $w = a^{N!} b a^{(N+1)!+1}$

w/ $w = xyz$ & $|xy| \leq N$ y must be some number of a's from the 1^{st} part of the string, say ~~x~~ is p a's i.e. $y = a^p$

Now to contradict the P.L. and prove L is not regular we need to show that for any $p \leq N$ the following:

$$\underbrace{N! + (i-1)p + 1}_{\text{for the } b} = \underbrace{(N+1)! + 1}_{\text{the extra a to balance the } b}$$ is true for some i

Solve the equation

$$N! + (i-1)p + \cancel{1} = (N+1)! + \cancel{1}$$

$$(i-1)p = (N+1)! - N! \quad \text{'pull out' an } N!$$

$$(i-1)p = N!(N+1-1) = N!N$$

$$i = \frac{N!N}{p} + 1$$ which is the i to make $w_i \notin L$ invalidating the P.L and proving L is not regular

4c) $L = \{a^n b^l a^k ;\ n = l \text{ or } l \neq k\}$

Note for a string not to be in L it must be the case that $n \neq l$ and $l = k$, so for the proof you need to find an i that makes $n \neq l$ and $l = k$

Let N be as in the P.L

Pick $w = a^N b^N c^N$ this is in L because $n = l$

now y must be some # of a's from be beginning of the string.

w_i (for any $i > 1$) will have $n \neq l$ because there are 'too many' a's and $l = k$ because it always was.

5a) answer in text

5b) $L = \{a^n :\ n \text{ is not prime}\}$ I'll be lazy and use proof-by-contradiction

Assume L is regular

Then, because the regular languages are closed under complementation $\bar{L}$ is regular,

But by problem 5a $\bar{L}$ is not regular, thus the assumption that L is regular must be false.

24. We know $L_1 \cup L_2$ is regular
and L_1 is regular

Is L_2 necessarily regular? No, although it might be.

Proof:
Let L_1 be all strings. E.g. over the alphabet $\{a, b\}$ $L_1 = L((a+b)^*) = L(M_1)$ where

M_1 is

Clearly L_1 is regular (I just wrote a regular expression and a DFA for it).

$L_1 \cup L_2 = L_1$ also regular
So, whether L_2 is regular or not
L_1 and $L_1 \cup L_2$ are.

26 $L = \{a^n b^m : n \geq 100 \ m \leq 50\}$

a) No. You can not use the P.L. to prove L is regular because the P.L. can only be used to prove that languages are not regular.

b) No. Because L is regular.

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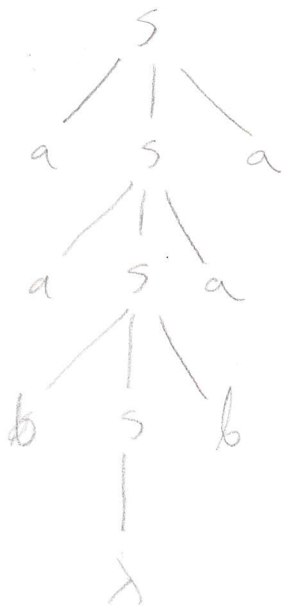
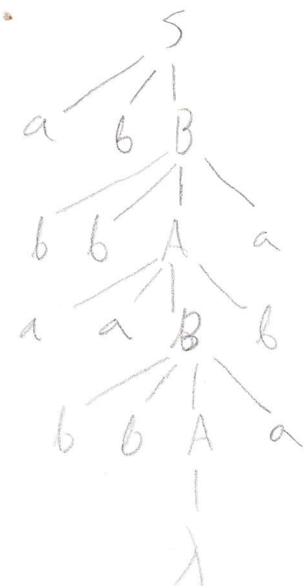


3. $w = abbbaabbaba$



Leftmost Derivation

$S \Rightarrow abB \Rightarrow abbbAa$

$\Rightarrow abbbaaBba$

$\Rightarrow abbbaabbAaba$

$\Rightarrow abbbaabbaba$

Note that there is only one variable per string so leftmost (& right most) is automatic.

7a) in text