

Homework #1

Pg. 13+

#8) Solution in back of book

#25) Show that

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

as in the class example for $\sum_{i=1}^n i$ this is best solved via an induction proof.

Base case: $n=1$

$$\frac{1(1+1)(2 \cdot 1 + 1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$$

$1^2 = 1$ and \checkmark

Induction assumption:

$$\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6}$$

Induction Step: Show (using the induction assumption) that

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\begin{aligned} \sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^k i^2 + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + \frac{6k^2 + 12k + 6}{6} \quad \text{by induction assumption} \\ &= \text{(next page)} \end{aligned}$$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6k^2+12k+6}{6}$$

$$= \frac{(k^2+k)(2k+1) + 6k^2+12k+6}{6}$$

$$= \frac{2k^3+3k^2+k+6k^2+12k+6}{6}$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

$$\cong \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \quad \checkmark$$

The first step in solving a system of linear equations is to write the equations in standard form. This means that each equation should be written as $Ax + By = C$, where A , B , and C are constants. Once the equations are in standard form, the next step is to choose a method for solving the system. There are three main methods: substitution, elimination, and graphing. Substitution involves solving one equation for one variable and then substituting that expression into the other equation. Elimination involves adding or subtracting the equations to eliminate one variable. Graphing involves plotting the equations on a coordinate plane and finding the point of intersection. The choice of method depends on the form of the equations and the preference of the solver.

Pg 27+

#2) In book

#4) In book

#9) Show that $(L^*)^* = L^*$ For all languages L

Formally $L^* = L^0 \cup L^1 \cup L^2 \dots$

$$\begin{aligned} \text{and } (L^*)^* &= (L^0 \cup L^1 \cup L^2 \cup \dots)^* \\ &= (L^0 \cup L^1 \cup L^2 \cup \dots)^0 \cup (L^0 \cup L^1 \cup L^2 \cup \dots)^1 \cup (L^0 \cup L^1 \cup L^2 \cup \dots)^2 \cup \dots \end{aligned}$$

One option is induction over the exponent

Slightly less formally we can say

for all $w \in (L^*)^N$ and all N it is true that $w \in L^*$

Thus $L^* = (L^*)^N$ for all N

So $L^* = (L^*)^*$

#12) $S \rightarrow aA$

$A \rightarrow bS$

$S \rightarrow \lambda$

For every aA combination there must be a bS combination.

Eg. $aA \Rightarrow abS$

So, every a is followed by b w/ the possibility of repeating ($S \rightarrow aA$ again) or stopping ($S \rightarrow \lambda$)

Note $S \rightarrow \lambda$ initially is possible so the empty string is included.

Thus,

$$L = \{(ab)^N : N \geq 0\}$$

Zero or more copies of 'ab'.