Homework #1

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#8) Solution in back of book

#25) Show that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

this is best solved via an induction as in the class example for $\sum_{i=1}^{n} i$ proof.

Base case: $n = 1$

$$\sum_{i=1}^{1} i^2 = 1 \text{ and } \frac{1(1+1)(2\cdot 1+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1 \quad \checkmark$$

Induction assumption:

$$\sum_{i=1}^{k} k^2 = \frac{k(k+1)(2k+1)}{6}$$

Induction Step: Show (using the induction assumption) that

$$\sum_{i=1}^{k+1} k^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\sum_{i=1}^{k+1} k^2 = \sum_{i=1}^{k} k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + k^2 + 2k + 1$$

by induction assumption

$$= \frac{k(k+1)(2k+1)}{6} + \frac{6k^2+12k+6}{6} = \text{(next page)}$$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6k^2+12k+6}{6}$$

$$= \frac{(k^2+k)(2k+1) + 6k^2+12k+6}{6}$$

$$= \frac{2k^3+3k^2+k+6k^2+12k+6}{6}$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \checkmark$$

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#2) In book

#4) In book

#9) Show that $(L^*)^* = L^*$ for all languages L

formally $L^* = L^0 \cup L^1 \cup L^2 \ldots$

and

$$(L^*)^* = (L^0 \cup L^1 \cup L^2 \cup \ldots)^*$$
$$= (L^0 \cup L^1 \cup L^2 \cup \ldots)^0 \cup (L^0 \cup L^1 \cup L^2 \cup \ldots)^1 \cup (L^0 \cup L^1 \cup L^2 \cup \ldots)^2 \cup \ldots$$

One option is induction over the exponent

Slightly less formally we can say

for all $w \in (L^*)^N$ and all N it is true that $w \in L^*$

Thus $L^* = (L^*)^N$ for all N

So $L^* = (L^*)^*$

#12) $S \to aA$ For every aA combination there must be a bS combination.

$A \to bS$ Eg. $aA \Rightarrow abS$

$S \to \lambda$ So, every a is followed by b w/ the possibility of repeating ($S \to aA$ again)

or stopping ($S \to \lambda$)

Note $S \to \lambda$ initially is possible so the empty string is included

Thus,

$$L = \{(ab)^N : N \geq 0\}$$

Zero or more copies of 'ab'