Signed Messages

- Traitors ability to lie makes Byzantine General Problem so difficult.
- If we restrict this ability, then the problem becomes easier
- Use authentication, i.e. allow generals to send unforgeable signed messages.

Signed Messages

- Assumptions about *Signed Messages*

A1: every message that is sent is delivered correctly

A2: the receiver of a message knows who send it

A3: the absence of a message can be detected

A4: a loyal general's signature cannot be forged, and any alteration of the contents of his signed messages can be detected. Anyone can verify the authenticity of a general's signature

Note: no assumptions are made about a traitor general, i.e. a traitor can forge the signature of another traitor.

Signed Messages

- Signed message algorithm assumes a *choice* function
  - if a set V has one single element v, then $choice(V) = v$
  - choice(Φ) = R, where Φ is the empty set
    - RETREAT is default
  - choice(A,R) = R
    - RETREAT is default
  - set V is <u>not</u> a multiset (recall definition of a multiset)
  - thus set V can have at most 2 elements, e.g. $V = \{A,R\}$.

Signed Messages

- Signing notation
  - let *v:i* be the value *v* signed by general *i*
  - let *v:i:j* be the message *v:i* counter-signed by general *j*
- each general *i* maintains his own set V_i containing all orders he received
- Note: do not confuse the set V_i of orders the general received with the set of all messages he received. Many different messages may have the same order.

BGP: Signed Message Solution

SM(m) -- from Lam82

Initially $V_i = \Phi$

1) The commander signs and sends his value to every lieutenant

2) For each *i*

A) If lieutenant *i* receives a message of the form v:0 from the commander and he has not yet received any order, then

i) he lets V_i equal {v}

ii) he sends the message $v{:}0{:}i$ to every other lieutenant

B) If lieutenant *i* receives a message of the form $v{:}0{:}j_1{:}\ldots{:}j_k$ and v is not in the set V_i, then

i) he adds v to V_i

ii) if $k<m$, then he sends the message $v{:}0{:}j_1{:}\ldots{:}j_k{:}i$ to every lieutenant other than $j_1,\ldots,j_k$

Algorithm SM(m)

- The SM(m) algorithm for signed messages works for

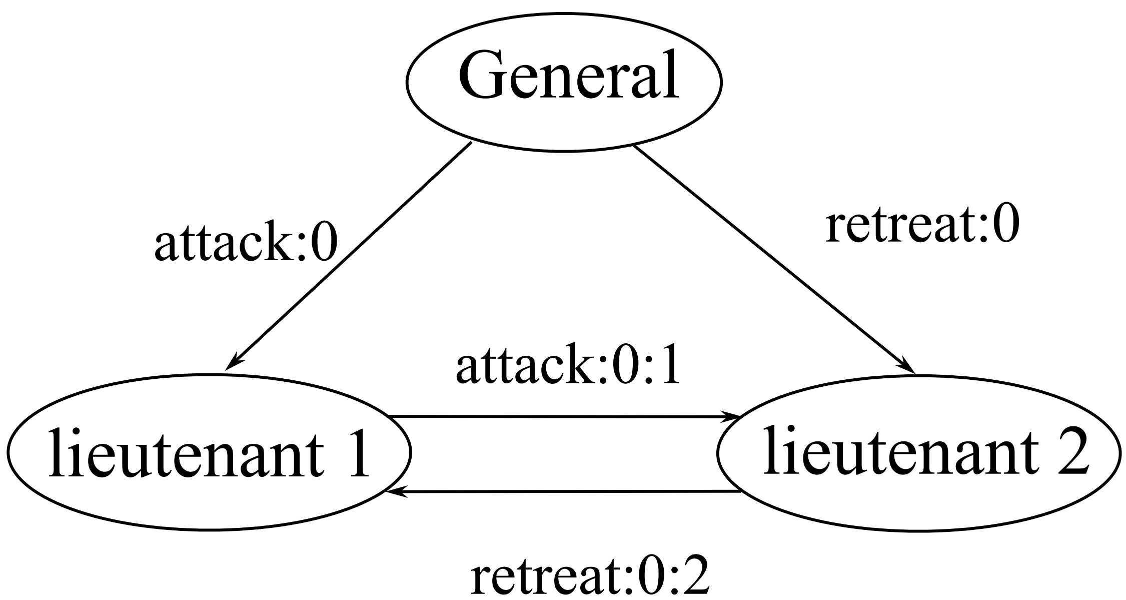
$$N \geq m + 2$$

– i.e. want non faulty commander and at least one non faulty lieutenant

- How does one know when one does not receive any more messages?
  - by *missing message assumption* A3, we can tell when all messages have been received
  - this can be implemented by using synchronized rounds
- Now traitor can be detected!
  - e.g. 2 correctly signed values => general is traitor

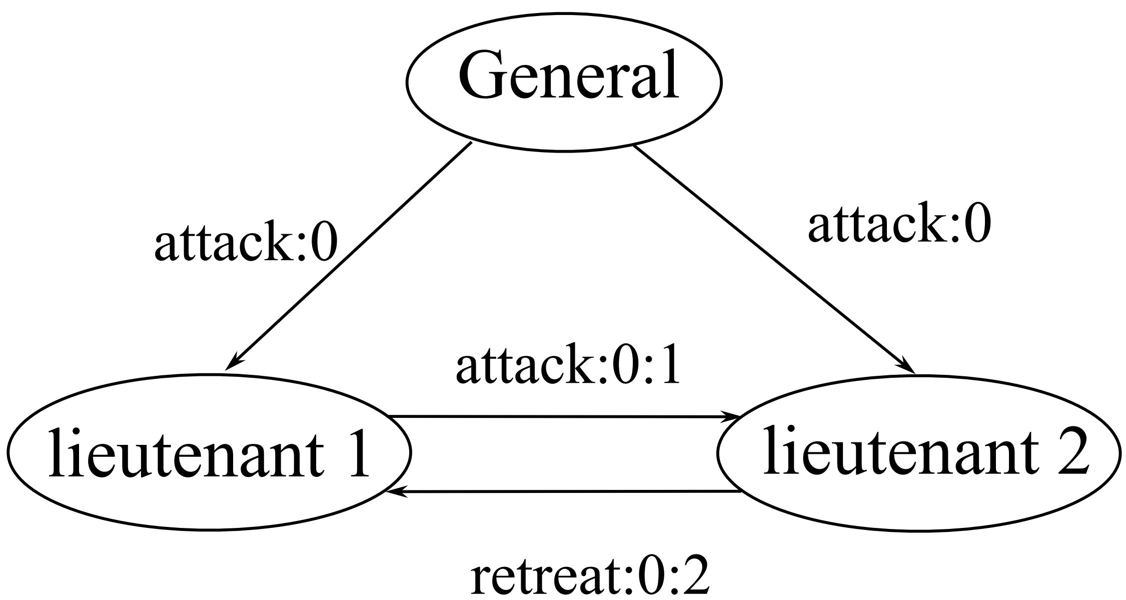
Algorithm SM(m)

- example, general is traitor



Algorithm SM(m)

- example, lieutenant 2 is traitor



Algorithm SM(m)

- example:
  - SM(0)
    - general sends *v:0* to all lieutenants
    - processor *i* receives *v:0* $V_i=\{v\}$
  - SM(1)
    - each lieut. countersigns and rebroadcasts *v:0*
    - processor *i* receives (*v:0:1, v:0:2,..., v:0:(N-1)*)

Algorithm SM(m)

- case 1: commander loyal, lieutenant j = traitor
  - all values except *v:0:j* are *v*

$$\Rightarrow v \in V_i \quad \forall \text{ loyal lieut. i}$$

  - processor *j* cannot tamper

$$\Rightarrow V_i = \{v\} \quad \forall \text{ loyal lieut. i}$$

- case 2: commander = traitor, => all lieut. loyal
  - all lieutenants correctly forward what they received
    - agreement: yes
    - validity: N/A

Algorithm SM(m)

- e.g.:
  - SM(2)
    - each lieut. countersigns and rebroadcasts all messages from the previous round
    - processor *i* has/receives
      - *v:0* — original message
      - *v:0:1, v:0:2, ... , v:0:(N-1)* — after 1st rebroadcast
      - ~~*v:0:1:1,*~~ *v:0:1:2, v:0:1:3, ..., v:0:1:N-1*

v:0:2:1, ~~*v:0:2:2,*~~ *v:0:2:3, ..., v:0:2:N-1*

...

v:0:N-1:1, v:0:N-1:2, v:0:N-1:3, ..., ~~*v:0:N-1:N-1*~~

after 2nd rebroadcast

Algorithm SM(m)

- case 1: commander loyal,   2 lieutenants are traitors
  - want each loyal lieut to get *V={v}*
  - round 0 => all loyal lieuts get *v* from commander
  - other rounds:
    - traitor cannot tamper
    - => all messages are *v* or Φ
- case 2: commander traitor + 1 lieut. traitor
  - round 0: all loyal lieuts receive *v:0*
  - round 1:
    - traitors send <u>one</u> value or Φ
  - round 2:
    - another exchange (in case traitor caused split in last round)
    - traitor still can <u>not</u> introduce new value

  => agreement: yes
  validity: N/A

Algorithm SM(m)

- Cost of signed message
  - encoding one bit in a code-word so faulty processor cannot "stumble" on it.
  - e.g.
    - » unreliability of the system $F_S = 10^{-10}/h$
    - » unreliability of single processor $F_P = 10^{-4}/h$
    - » want: Probability of randomly generated valid code word

$$P = \frac{10^{-10}}{10^{-4}} = 10^{-6} \approx 2^{-20}$$

    - » given 2^i valid codewords, want (20+i) bits/signature
    - » e.g. Attack/Retrieve

=> 2^1

=> 21 bit signature

Agreement

- Important notes:
  - there is no way to guarantee that different processors will get the same value from a possibly faulty input device, except having the processors communicate among themselves to solve the Byz.Gen. Problem.
  - faulty input device may provide meaningless input values
    - » all that Byz.Gen. solution can do is guarantee that all processors use the same input value.
    - » if input is important, then use redundant input devices
    - » redundant inputs cannot achieve reliability. It is still necessary to insure that all non-faulty processors use the redundant data to produce the same output.

Agreement

- Implementing BGP is no problem
- The problem is implementing a message passing system that yields respective assumptions, i.e.:

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A2: the receiver of a message knows who send it

A3: the absence of a message can be detected

A4: a loyal general's signature cannot be forged, and any alteration of the contents of his signed messages can be detected. Anyone can verify the authenticity of a general's signature