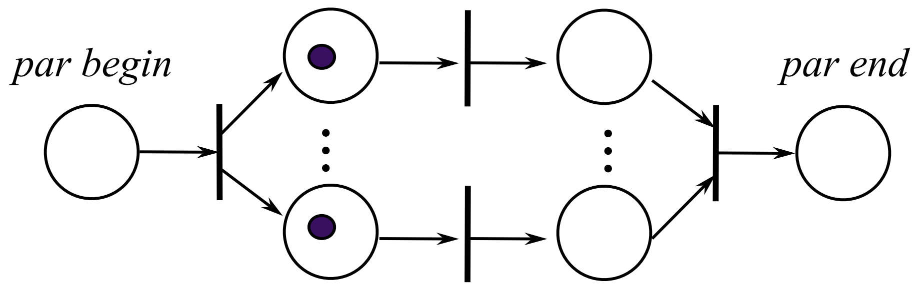
Petri Nets

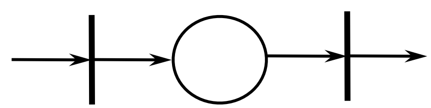
- Definitions
  - **Source Transition:** a transition without any input place
    - » is unconditionally enabled
  - **Sink Transition:** a transition without any output place
    - » consumes but does not create any tokens
  - **Self-Loop:** *P* is both an input and output place of *T*
  - **Pure Petri Net:** does not contain self-loops
  - **Ordinary Petri Net:** all of the arc weights are unity, i.e. one.
  - **Infinite Capacity Net:** assumes that each place can accommodate an unlimited number of tokens
  - **Finite Capacity Net:** max. token-capacity *K(P)* defined for each *P*
  - **Strict Transition Rule:** finite capacity net with additional rule that the number of tokens in each output place *P* of *T* cannot exceed its capacity *K(P)* after firing *T*.

Petri Nets

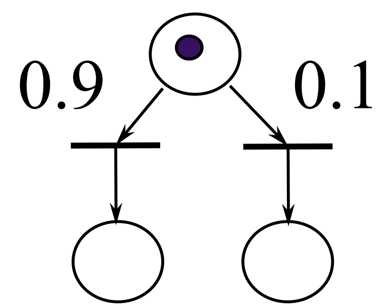
- Modeling Constructs
  - Concurrency



  - Precendence

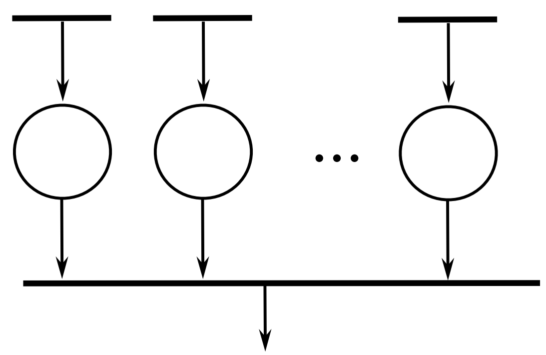


  - Conflict, choice or decision
    - function: “exclusive OR”
    - only one transition can fire
    - weight: probability of taking that arc



Petri Nets

- Modeling Constructs
  - Synchronization
    - AND
    - joining several paths into a single path



Example

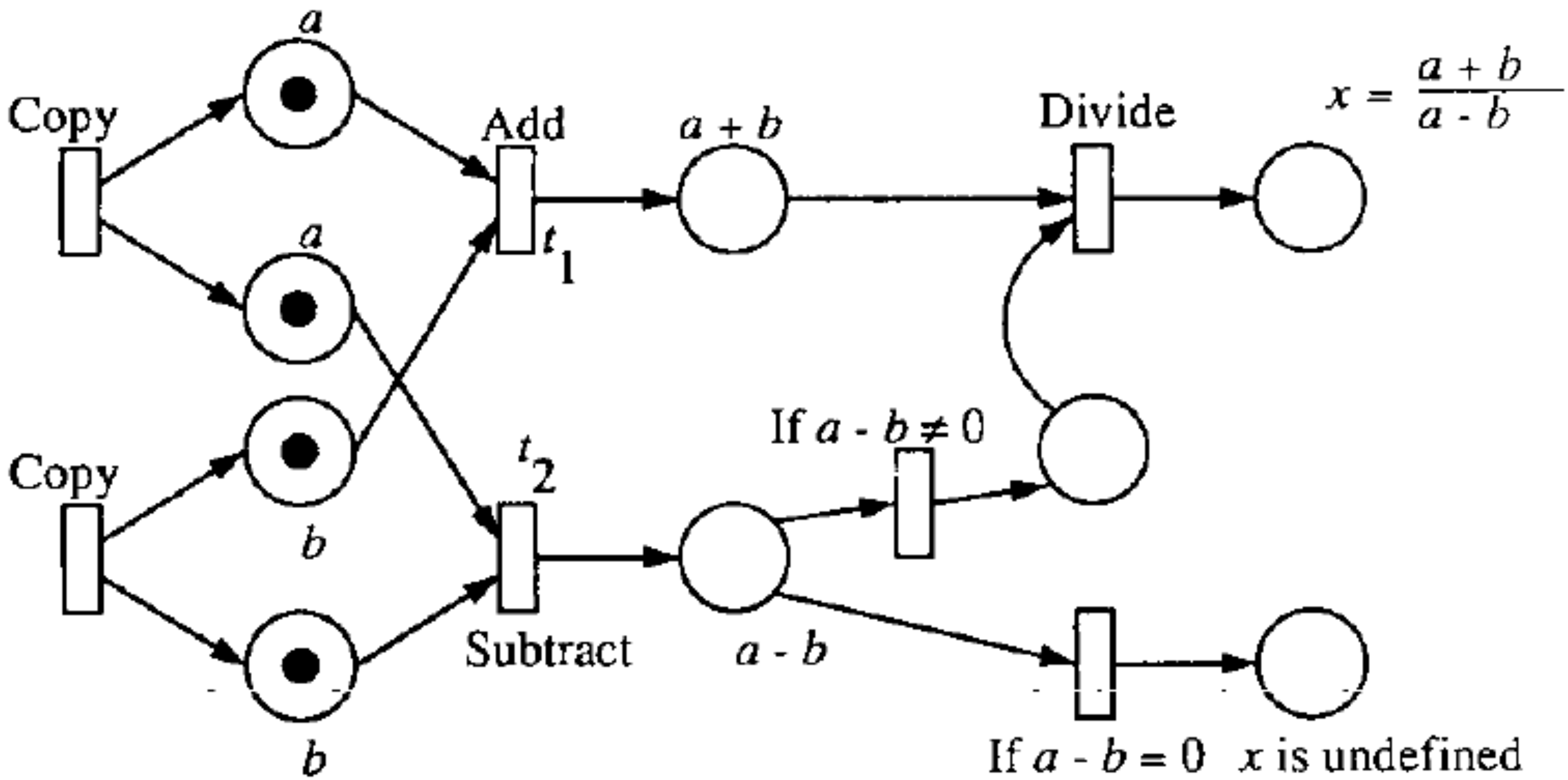


Fig. 8. A Petri net showing a dataflow computation for $x = (a + b)/(a - b)$.

Example

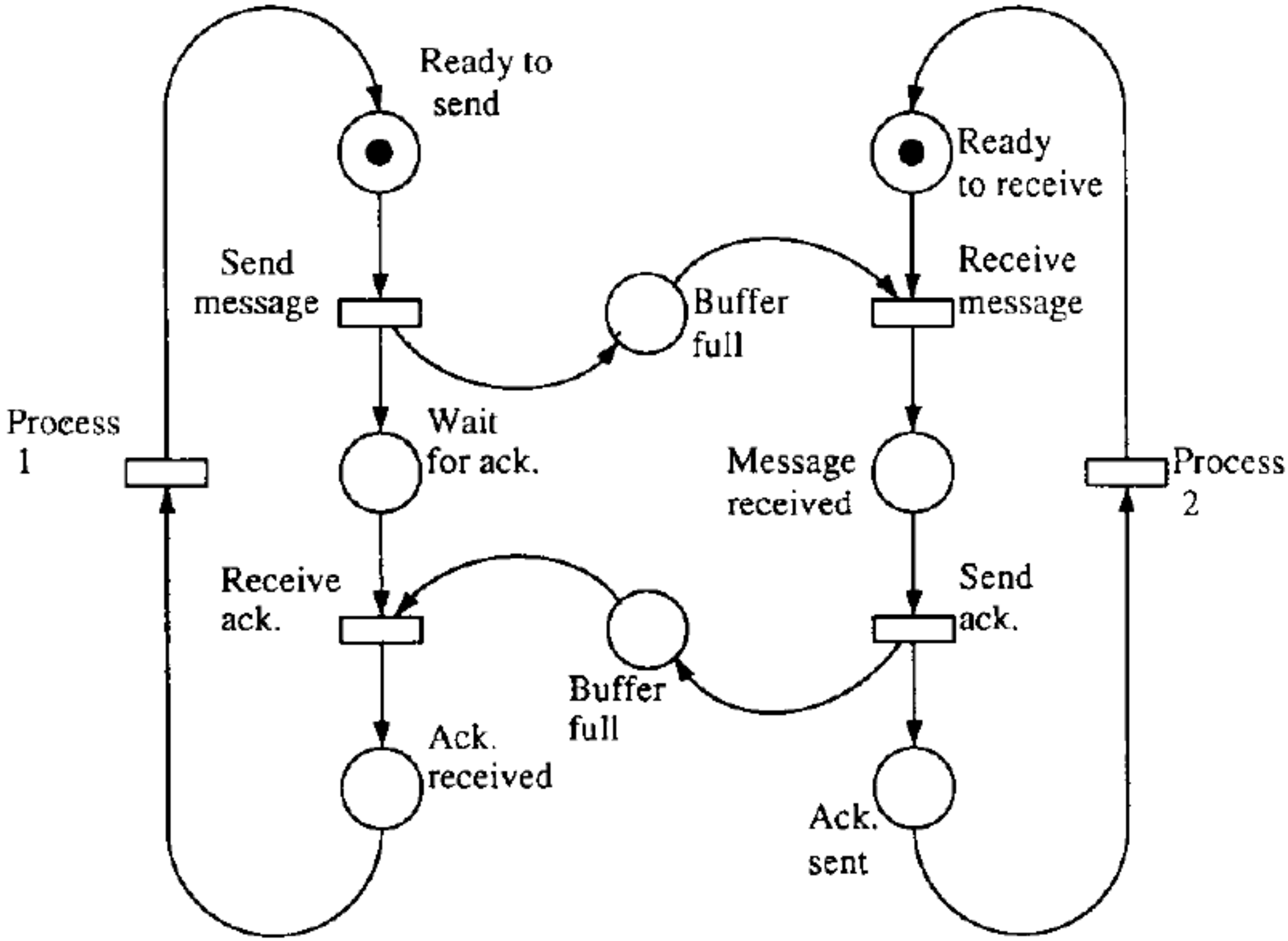
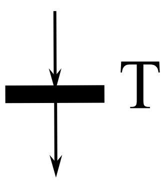


Fig. 9. A simplified model of a communication protocol.

Petri Nets

- Modeling Constructs
  - Time
    - need new concept => timed transition
    - timed transition has firing delay T
    - when transition is enabled, wait T, then fire
      - tokens are consumed and created at the firing instance
    - timed Petri Net symbol



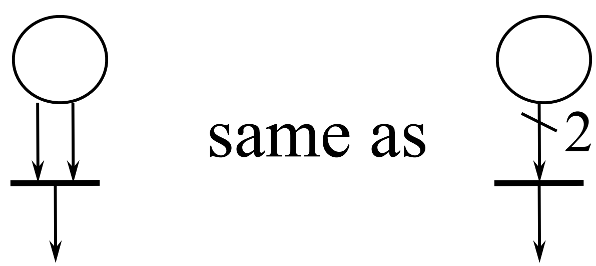
- Stochastic Petri Net
  - T is not fixed
  - T = random variable with *exponential distribution*

Petri Nets

- Generalized Stochastic Petri Nets (GSPN)

Adds extra constructs

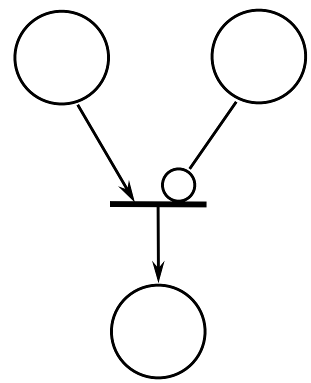
- Mixed transitions
  - stochastic and instantaneous transitions
- Multiple Arcs



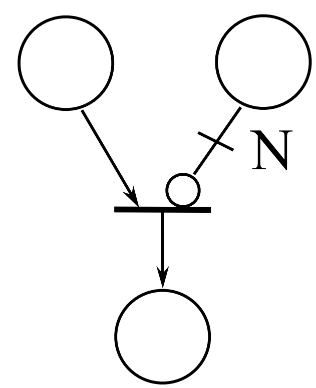
  - needs 2 tokens to fire

Petri Nets

- Generalized Stochastic Petri Nets (cont.)
  - Inhibitory Arcs
    - token inhibits firing
    - obviously no token transfer
    - watch for deadlocks!



  - Multiple Inhibitory Arcs
    - needs at least N tokens to inhibit firing
    - less than N tokens => transition is firable



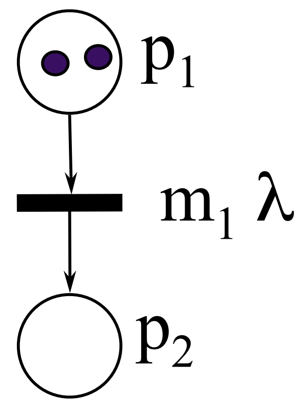
Petri Nets

- Reachability
  - fundamental basis for studying the dynamic properties of any system
  - firing of enabled transition will change token distribution
  - sequence of firings results in sequence of markings
  - marking M_n is reachable from M_0 if there exists a sequence of firings that transforms M_0 into M_n
  - firing sequence is denoted by
    - » $\sigma = M_0\ t_1\ M_1\ t_2\ \ldots\ t_n$ or simply $\sigma = t_1\, t_2 \ldots t_n$
    - » in this case M_n is reachable from M_0 by σ
  - the set of all possible markings reachable from M_0 in a net (N,M_0) is denoted by $R(N,M_0)$ or simply $R(M_0)$
  - the set of all possible firing sequences from M_0 in a net (N,M_0) is denoted by $L(N,M_0)$ or simply $L(M_0)$

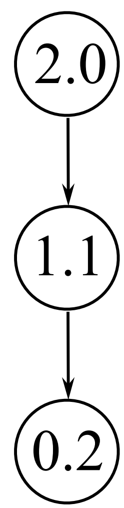
Petri Nets

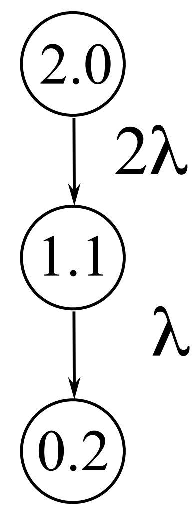
- Reachability Graph
  - Petri Net with initial marking

    $$M(t_0) = \{m_1, m_2\} = \{2,0\}$$



  - Reachability Graph
    - » add transitions to graph and…
    - » Markov chain

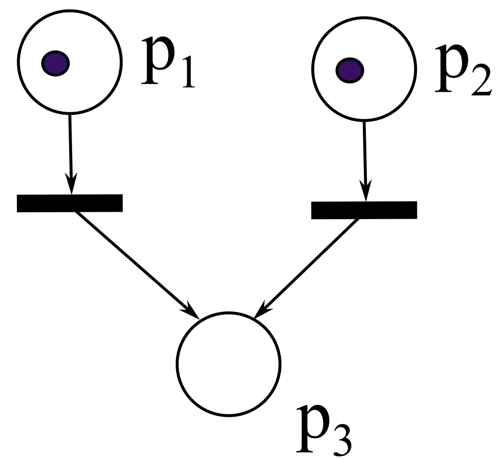




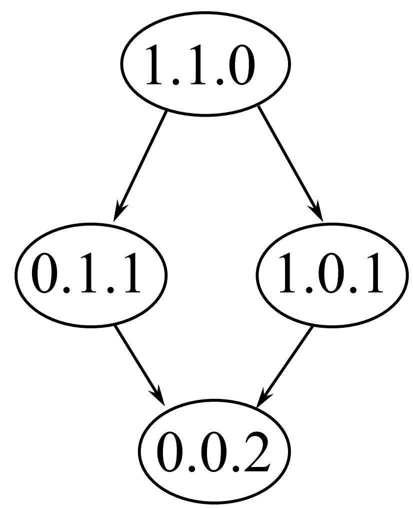
Petri Nets

- Reachability Graph
  - Petri Net with initial marking

$$M(t_0) = \{m_1, m_2, m_3\}$$

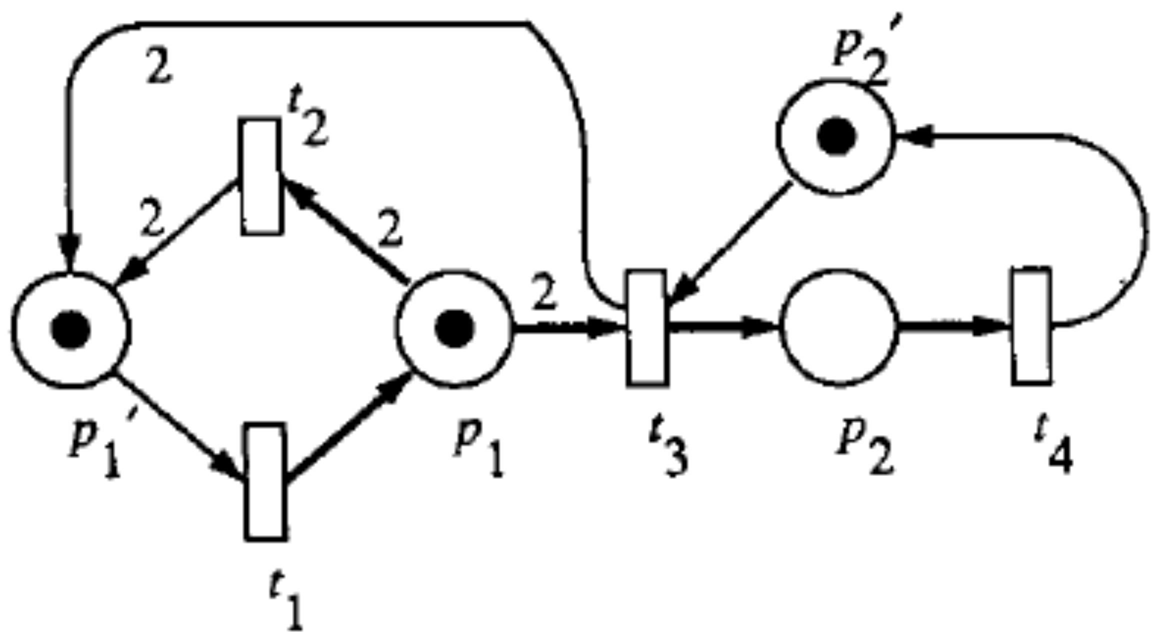


  - Reachability Graph



Petri Nets

- Boundedness
  - A Petri net (N,M_0) is said to be *k-bounded* (or simply *bounded*) if the number of tokens in each place does not exceed a finite number k of any marking reachable from M_0 , i.e., $M(p) \leq k$ for every place p and every marking $M \in R(M_0)$
  - example of 2-*bound* net

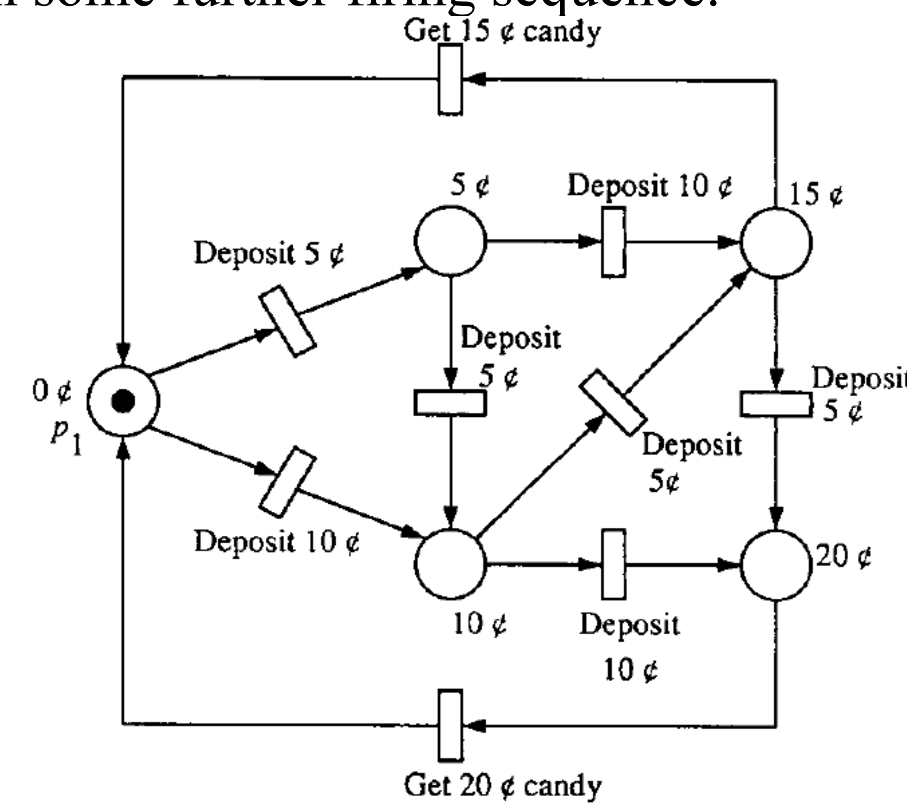


Petri Nets

- Liveness
  - closely related to the complete absence of deadlock in OS
  - A Petri net (N,M_0) is said to be *live* (or equivalently M_0 is said to be a *live* marking of N) if, no matter what marking has been reached from M_0, it is possible to ultimately fire *any* transition of the net by progressing through some further firing sequence.

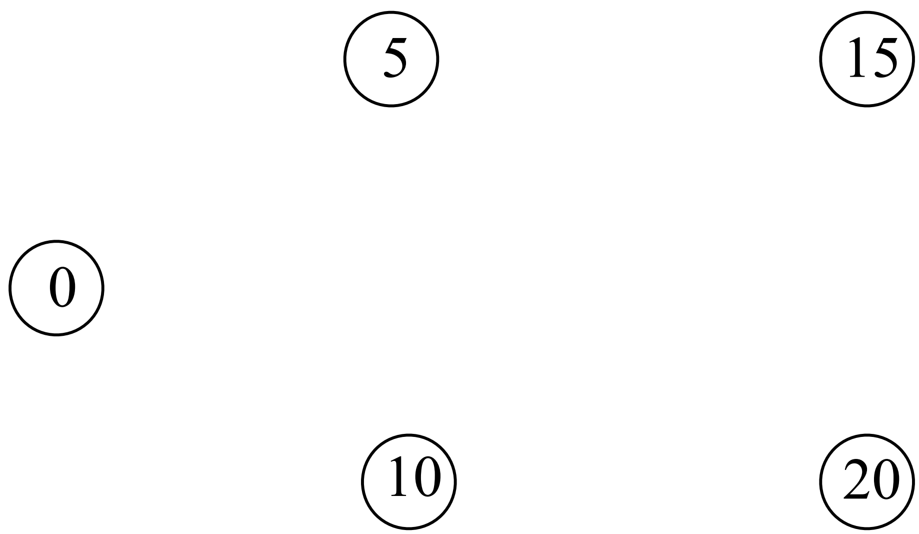
A live Petri net guarantees deadlock-free operation, no matter what firing sequence is chosen.

However, this property is costly to verify, e.g. for large systems.



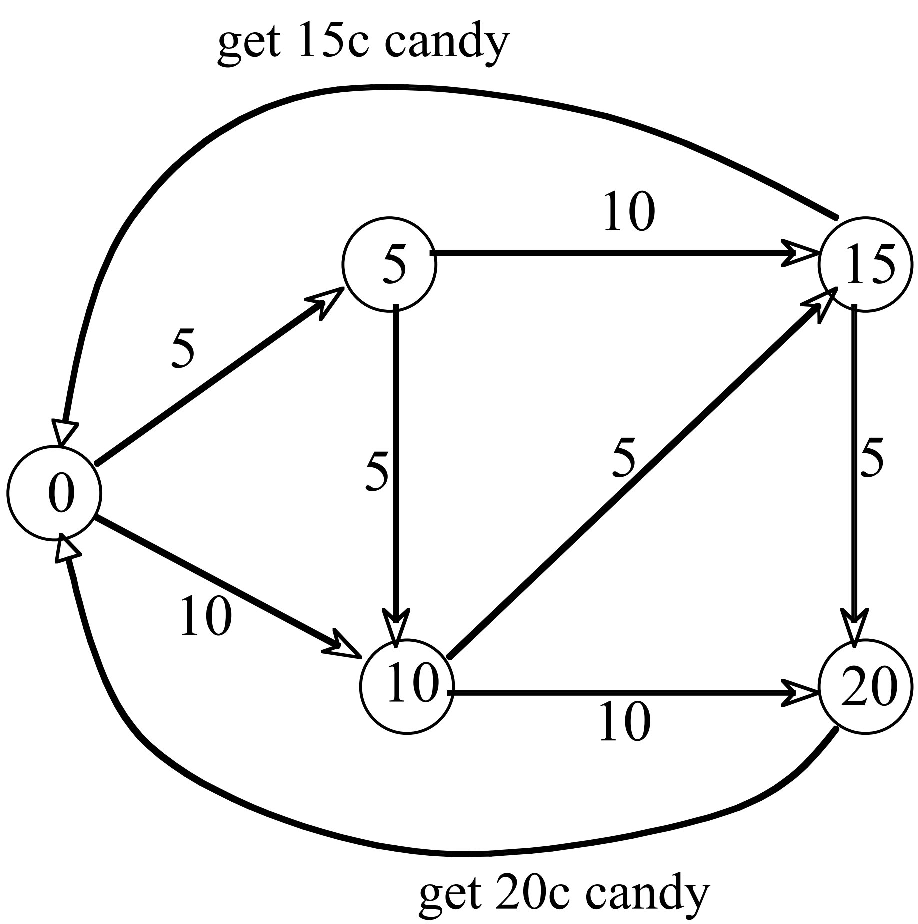
Petri Nets

- How did we get the net of the candy machine?
  - identify places needed



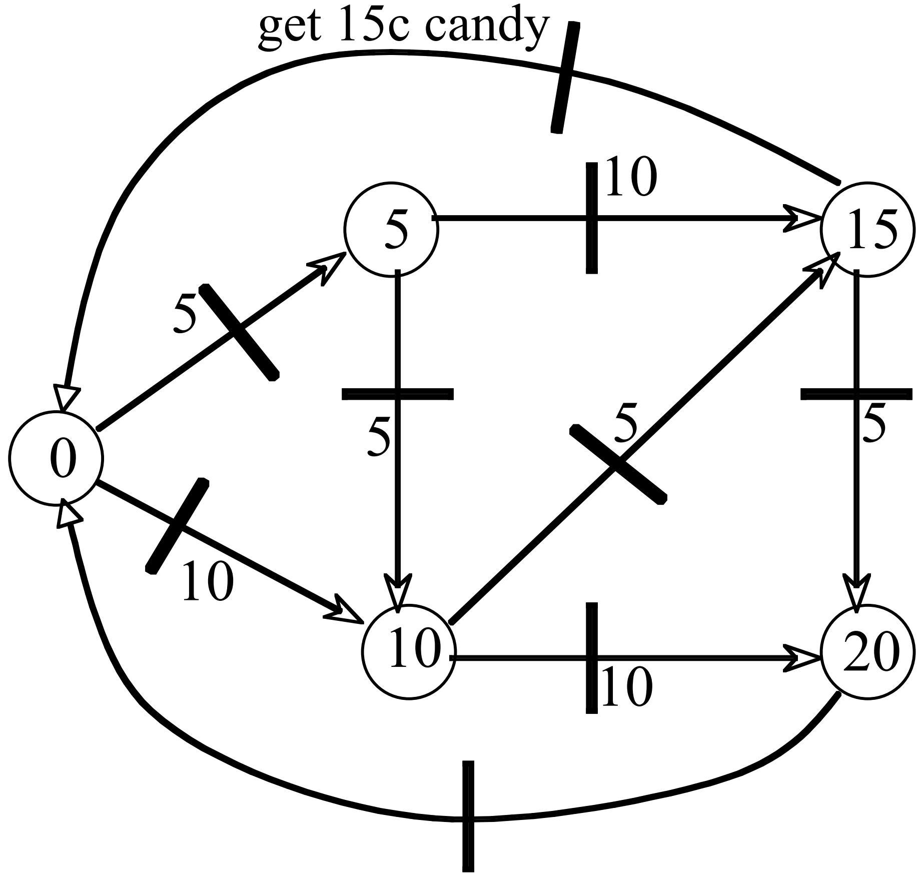
Petri Nets

- Example: candy machine
  - identify paths from places to places and the events that get you there (interpret the numbers as "deposit x cents".



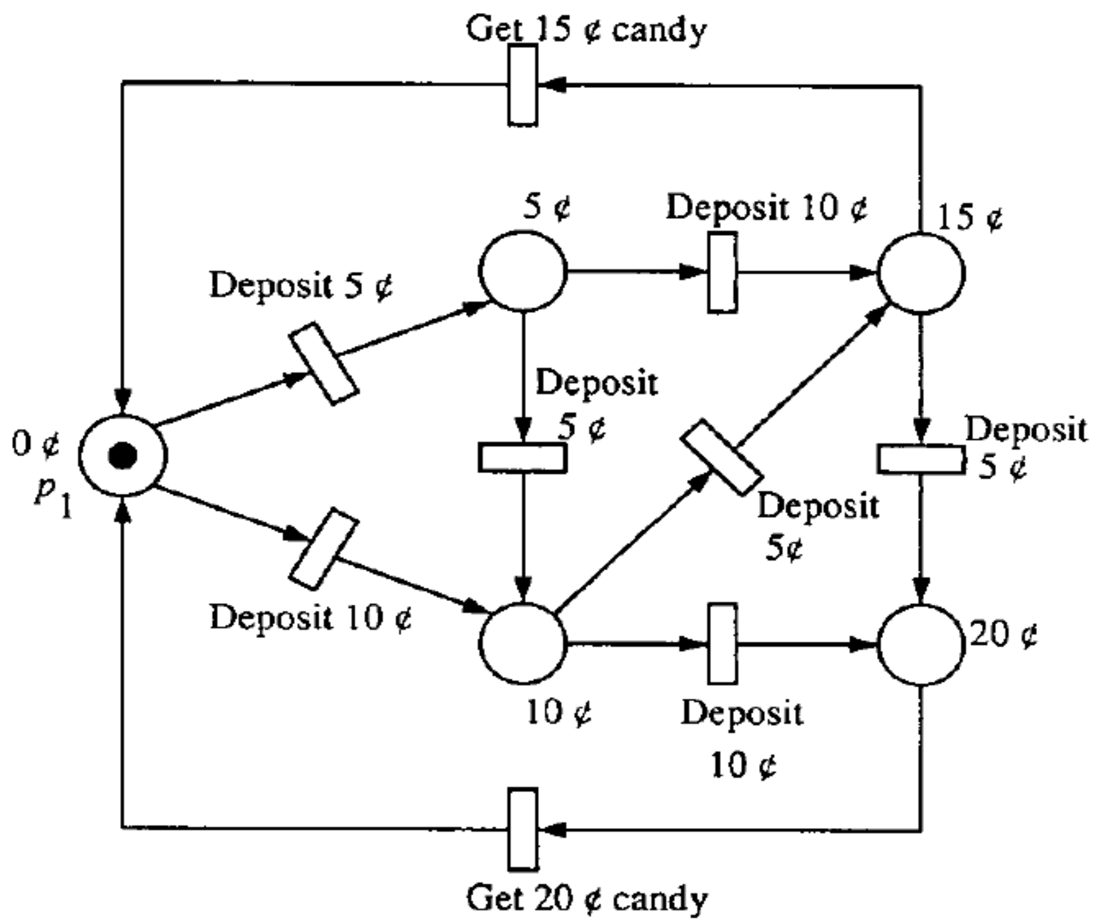
Petri Nets

- Example: candy machine
  - transition events: "deposit x cents"



Petri Nets

- Example: candy machine
  - final Petri net



GSPN

- gspn model name (opt. param. list) (See language description)
  - 1. List all places and initial marking
    - place-name expr for init num of tokens
  - 2. List all timed trans. and rates
    - trans-name ind expr for rate
    - trans-name dep place-name expr for base rate
  - 3. List instant. trans. and branch weights
    - trans-name ind expr for weight
    - trans-name dep place-name expr for base weight
  - 4. List all place to trans. arcs
    - place-name trans-name expr for mult.
  - 5. List all trans. to place arcs
    - trans-name place-name expr for mult.
  - 6. List all inhibitory arcs

GSPN

- Some general notes
  - Recall: reachability graph is Markov.
  - Most functions compute CDF of “time to absorption” in reachability graph.
  - Must ensure net is “dead" at desired point, e.g.:
    - when 1st token enters “Failure" place,
    - when exactly k-of-N nodes are faulty,
    - when exactly k-of-N nodes are still up,
  - Need Inhibitory arcs from “Failure” back to **all** timed transitions.
    - Causes net to become dead at instant of failure.
    - Otherwise absorption could occur well after failure.

GSPN

- Useful Functions
  - etokt (t; model name, place-name {; args})
    - Expected num of tokens in place at time t.
  - etok (model name, place-name {; args})
    - Steady state average of same thing (no t parameter).
  - premptyt (t; model name, place-name {; args})
    - Probability place is empty at time t,
    - Useful for tracking failure modes,
    - Warning: Do not use (1- premptyt) !!!
  - prempty (model name, place-name {; args})
    - Steady state average of same thing (no t parameter).

GSPN

- Useful Functions
  - tput, tputt, taveputt
    - Difference is point-in-time of analysis.
    - Function:
      - The “throughput” of a transition
      - The “firing rate” of the transition
    - More useful in Performance models (jobs/sec).
    - tput: throughput for transition
    - tputt: throughput for transition at time t
    - taveputt: time-averaged throughput of a transition during interval (0,t)

GSPN

- Useful Functions
  - util, utilt, taveutil
    - Difference is point-in-time of analysis
    - Function:
      - The "utilization" of a timed transition
      - The fraction of time it is enabled.
      - Also useful in Performance models (proc. util).
    - util: utilization for a transition
    - utilt: utilization for a transition at time t

GSPN Example

- K-of-N System: Model A

```
* SYSTEM: K of N SYSTEM. ALTERNATE MODEL DEMONSTRATION
* MODELS: GSPN

epsilon results 1.0*10^(-11)
epsilon basic   1.0*10^(-13)
format 3

*------------------------- MODEL DEFINITION -- MODEL A
gspn KofN_A (K,N)
*
* 1. INITIAL MARKING M(0) ..................................... P_NAME TOKENS
n_up  N
n_dn  0
end
*
* 2. TIMED TRANSITIONS ........... T_NAME ind RATE (or) T_NAME dep P_NAME RATE
flt dep n_up lambda
end
*
* 3. INSTANT. TRANSITIONS .... T_NAME ind WEIGHT (or) T_NAME dep P_NAME
WEIGHT
end
*
* 4. PLACE - TRANS ARCS .................................. P_NAME T_NAME MULT
n_up flt 1
end
*
* 5. TRANS - PLACE ARCS .................................. T_NAME P_NAME MULT
flt n_dn 1
end
*
* 6. INHIBITORY ARCS ..................................... P_NAME T_NAME MULT
n_dn flt  (N-K+1)
end
```

GSPN Example

- K-of-N System: Model B

```
*------------------------- MODEL DEFINITION -- MODEL B
gspn KofN_B (K,N)
*
* 1. INITIAL MARKING M(0) ...................................... P_NAME TOKENS
n_up     N
n_dn     0
SYS_FAIL 0
end
*
* 2. TIMED TRANSITIONS ........... T_NAME ind RATE (or) T_NAME dep P_NAME RATE
flt dep n_up lambda
end
*
* 3. INSTANT. TRANSITIONS .... T_NAME ind WEIGHT (or) T_NAME dep P_NAME WEIGHT
fail_sys  ind  1
end
*
* 4. PLACE - TRANS ARCS .................................... P_NAME T_NAME MULT
n_up flt 1
n_dn fail_sys  (N-K+1)
end
*
* 5. TRANS - PLACE ARCS .................................... T_NAME P_NAME MULT
flt n_dn 1
fail_sys  SYS_FAIL  1
end
*
* 6. INHIBITORY ARCS ...................................... P_NAME T_NAME MULT
SYS_FAIL  flt  1
end
```

GSPN Example

- K-of-N System: Model C

```
*------------------------- MODEL DEFINITION -- MODEL C
gspn KofN_C (K,N)
*
* 1. INITIAL MARKING M(0) ...................................... P_NAME TOKENS
n_up     N
n_dn     0
sys_up   1
SYS_FAIL 0
end
*
* 2. TIMED TRANSITIONS ........... T_NAME ind RATE (or) T_NAME dep P_NAME RATE
flt dep n_up lambda
end
*
* 3. INSTANT. TRANSITIONS .... T_NAME ind WEIGHT (or) T_NAME dep P_NAME WEIGHT
fail_sys ind 1
end
*
* 4. PLACE - TRANS ARCS .................................. P_NAME T_NAME MULT
n_up    flt      1
sys_up  fail_sys 1
end
*
* 5. TRANS - PLACE ARCS .................................. T_NAME P_NAME MULT
flt      n_dn     1
fail_sys SYS_FAIL 1
end
*
* 6. INHIBITORY ARCS ..................................... P_NAME T_NAME MULT
n_up      fail_sys  K
SYS_FAIL  flt       1
end
```