Stand-by Redundancy

- When primary component fails, standby component is started up.
- Stand-by spares are cold spares => unpowered
- Switching equipment assumed failure free

Let X_i denote the lifetime of the i-th component from the time it is put into operation until its failure.

System lifetime:

$$X_{sys} = \sum_{i=1}^{n} X_i$$

Stand-by Redundancy

- MTTF $$E(X) = \frac{n}{\lambda}$$
  - gain is linear as a function of the number of components, unlike the case of parallel redundancy
  - added complexity of detection and switching mechanism

M-of-N System

Starting with N components, we need any M components operable for the system to be operable.

Example: TMR

$$R_{\mathrm{TMR}}(t) = R_1(t)R_2(t)R_3(t) + R_1(t)R_2(t)(1 - R_3(t)) + R_1(t)(1 - R_2(t))R_3(t) + (1 - R_1(t))R_2(t)R_3(t)$$

Where $R_i(t)$ is the reliability of the i-th component

if $R_i(t) = R_1(t) = R_2(t) = R_3(t) = R(t)$ then

$$\begin{aligned} R_{\mathrm{TMR}}(t) &= R^3(t) + 3R^2(t)(1 - R(t)) \\ &= R^3(t) + 3R^2(t) - 3R^3(t) \\ &= 3R^2(t) - 2R^3(t) \end{aligned}$$

M-of-N System

The probability that exactly *j* components are not operating is

$$\binom{N}{j} Q^{j}(t) R^{N-j}(\mathrm{t}) \qquad \text{with} \binom{N}{j} = \frac{N!}{j!(N-j)!}$$

then

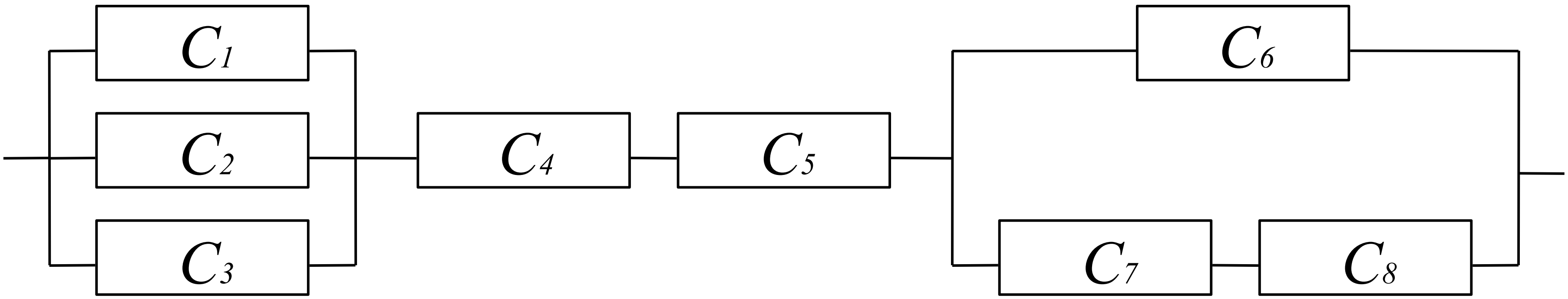
$$R_{MofN}(t) = \sum_{i=0}^{N-M} \binom{N}{i} Q^{i}(t) R^{N-i}(t)$$

Reliability Block Diagram

- Series Parallel Graph
  - a graph that is recursively composed of series and parallel structures.
  - therefore it can be "collapsed" by applying series and/or parallel reduction
  - Let C_i denote the condition that component *i* is operable
    - » 1 = up, 0 = down
  - Let *S* denote the condition that the system is operable
    - » 1 = up, 0 = down
- *S* is a logic function of *C*'s

Reliability Block Diagram

- Example:



$$S = (C_1 + C_2 + C_3)(C_4 C_5)(C_6 + C_7 C_8)$$

+ => parallel (1 of N)

. => series (N of N)

K of N system

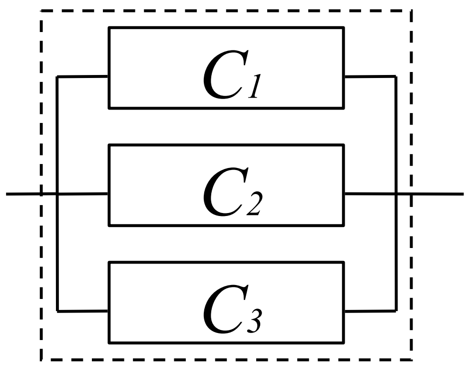
- Example 2-of-3 system

$$S = (C_1C_2 + C_1C_3 + C_2C_3)$$

may abbreviate

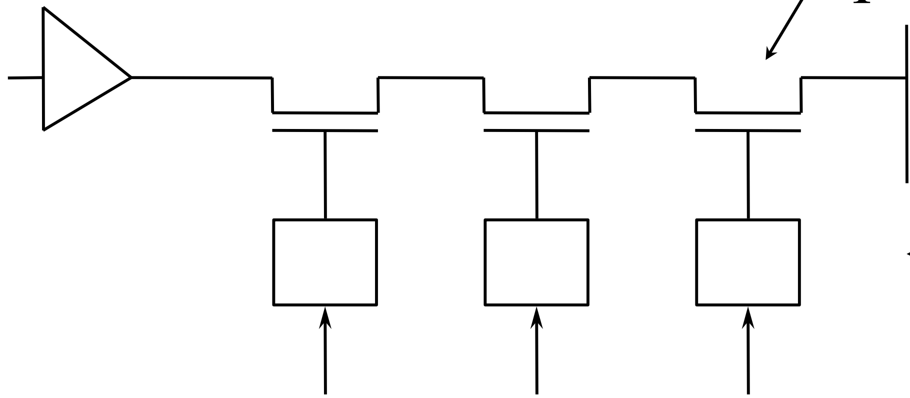
$$S = {}^2\!/_3(C_1C_2C_3)$$

draw as parallel



2-of-3

Example: Bus-Guardian



- assume λ for transistor & logic $\lambda = 2 \times 10^{-5}$
- 50/50 split: fail-on/fail-off

Two failure states for system

- Q_A = failed active (babbling) with λ_A
- Q_P = failed passive with λ_P

Example: Bus-Guardian

$$\lambda = 2 \times 10^{-5}$$

$$\lambda_A = 1 \times 10^{-5}$$

$$\lambda_P = 1 \times 10^{-5}$$

$$MTTF = \frac{1}{\lambda} = 5 \times 10^{4}$$

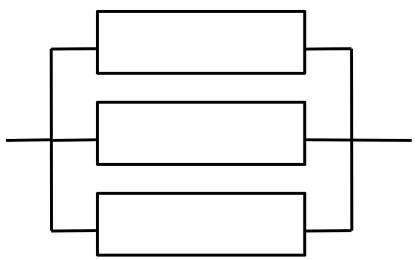
$$MTTF_A = \frac{1}{\lambda_A} = 10^{5}$$

$$MTTF_P = \frac{1}{\lambda_P} = 10^{5}$$

for each stage

Example: Bus-Guardian

- Active Failure
  - if any one bus guardian is correct then no babble possible
  - thus we use 1-of-N parallel system model



$$Q(t) = \prod_{i=1}^{3} Q_i(t)$$

with $$Q_i(t) = 1 - e^{-\lambda_A t}$$

Example: Bus-Guardian

- Solution - Parallel
  - if any one bus guardian is correct then no babble possible
  - 1-of-N parallel system model

$$Q(t) = (1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})$$
$$= 1 - 3e^{-\lambda_A t} + 3e^{-2\lambda_A t} - e^{-3\lambda_A t}$$

$$\text{e.g. with } \lambda_A = 10^{-5}/h \text{ and } t = 1000h$$
$$\lambda_A t = 0.01$$

Example: Bus-Guardian

compute: $Q(t) = 1 - 3e^{-\lambda_A t} + 3e^{-2\lambda_A t} - e^{-3\lambda_A t}$

$$Q(1000h) = 1 - 3(0.9900498) + 3(0.9801987) - (0.9704455)$$
$$= 1.2 \times 10^{-6}$$

compute:

$$Q(t) = (1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})$$
$$= (1 - e^{-\lambda_A t})^3$$

$$Q(1000h) = 0.9851243 \times 10^{-6}$$

in general: danger of cancellation
=> catastrophic results,
=> legal issues (even though one should realize what the fail rates really mean)

Example: Bus-Guardian

$$\text{MTTF}_\text{A} = \int_0^\infty R(t)dt = \int_0^\infty 1 - Q(t)dt$$

$$= \int_0^\infty (3e^{-\lambda_A t} - 3e^{-2\lambda_A t} + e^{-3\lambda_A t})dt$$

$$= \left[-\frac{3}{\lambda_A} e^{-\lambda_A t} + \frac{3}{2\lambda_A} e^{-2\lambda_A t} - \frac{1}{3\lambda_A} e^{-3\lambda_A t} \right]_0^\infty$$

simplification:

$e^{-\lambda_A t} = 0$ as $t \rightarrow \infty$

$e^{-\lambda_A t} = 1$ with $t = 0$

$$\text{MTTF}_\text{A} = \frac{3}{\lambda_A} - \frac{3}{2\lambda_A} + \frac{1}{3\lambda_A}$$

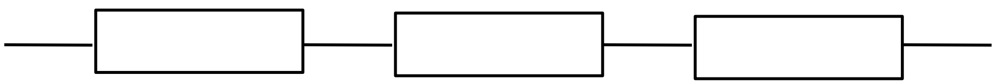
$$= (3 - \frac{3}{2} + \frac{1}{3}) \times 10^5$$

$$= 1.83 \times 10^5 h$$

3 drivers result in approx. MTTF of twice and not three times that of single driver

Example: Bus-Guardian

- Passive Failure
  - any one of *N* bus guardians can take out subsystem
  - thus we use series system model



$$R(t) = \prod_{i=1}^{3} R_i(t)$$

$$= e^{-\sum_{i=1}^{3} \lambda_i t}$$

$$= e^{-3\lambda t}$$

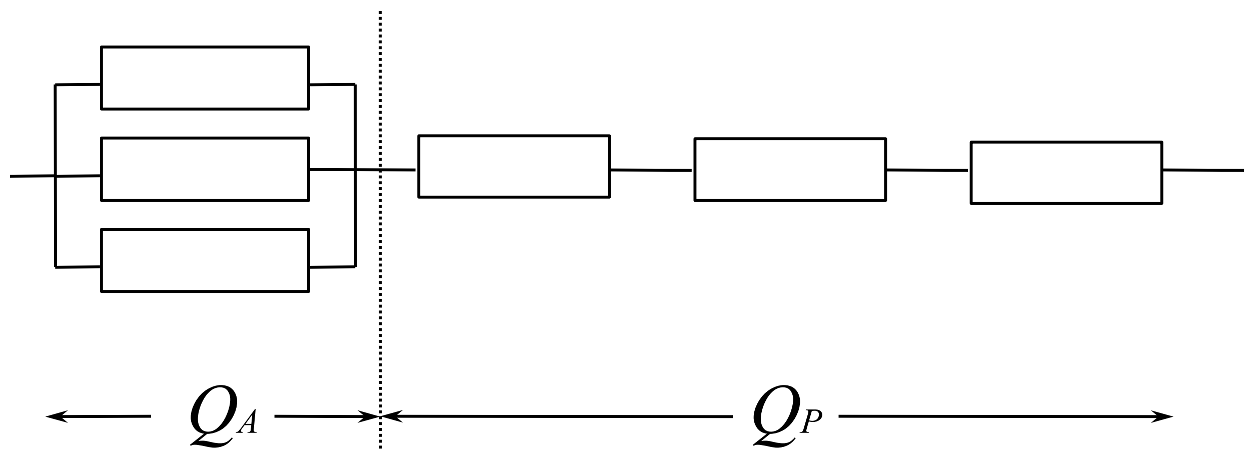
Given $\lambda = 1 \times 10^{-5}$ $\quad t = 1000\text{h}$

$$R(t) = e^{-3\lambda t} = 0.9704455$$

$$\Rightarrow MTTF = \frac{1}{\lambda_{sysp}} = 33333h$$

Example: Bus-Guardian

- summary
  - active failure => parallel => Q_A
  - passive failure => series => Q_P
  - whole system fails if either mode occurs => series



Example: Bus-Guardian

- summary

	Simplex	Triplex
$MTTF_A$	$1\times10^5 h$	$1.8\times10^5 h$
$MTTF_P$	$1\times10^5 h$	$0.33\times10^5 h$
$MTTF$	$0.5\times10^5 h$	$0.28\times10^5 h$

$$MTTF = \frac{MTTF_A \times MTTF_P}{MTTF_A + MTTF_P}$$

What is the unreliability Q_A?

- Two approaches to compute Q(t) at 1000h

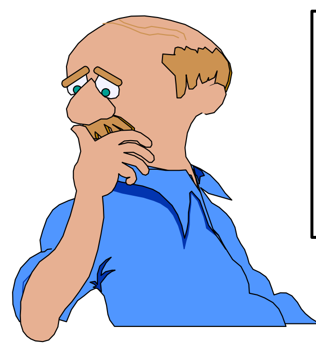
1) $$Q(t) = (1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})(1 - e^{-\lambda_A t})$$
$$= 1 - 3e^{-\lambda_A t} + 3e^{-2\lambda_A t} - e^{-3\lambda_A t}$$

2) $$MTTF_A = 1.8333 \times 10^5$$

using $MTTF = \frac{1}{\lambda}$ we compute λ and use

$$Q(t) = (1 - e^{-\lambda t})$$

Now we compute Q(1000) and ...



What is wrong?