

Markov Process

- ◆ A stochastic process is a function whose values are random variables
- ◆ The classification of a random process depends on different quantities
 - state space
 - index (time) parameter
 - statistical dependencies among the random variables $X(t)$ for different values of the index parameter t .

Markov Process

- ◆ State Space
 - the set of possible values (states) that $X(t)$ might take on.
 - if there are finite states => *discrete-state process* or *chain*
 - if there is a continuous interval => *continuous process*
- ◆ Index (Time) Parameter
 - if the times at which changes may take place are finite or countable, then we say we have a *discrete-(time) parameter process*.
 - if the changes may occur anywhere within a finite or infinite interval on the time axis, then we say we have a *continuous-parameter process*.

Markov Process

- ◆ In 1907 A.A. Markov published a paper in which he defined and investigated the properties of what are now known as Markov processes.
- ◆ A Markov process with a discrete state space is referred to as a *Markov Chain*
- ◆ A set of random variables forms a Markov chain if the probability that the next state is $S_{(n+1)}$ depends only on the current state $S_{(n)}$, and not on any previous states

Markov Process

- ◆ States must be
 - mutually exclusive
 - collectively exhaustive
- ◆ Let $P_i(t)$ = Probability of being in state S_i at time t .

$$\sum_{\forall i} P_i(t) = 1$$

- ◆ Markov Properties
 - future state prob. depends only on current state
 - » independent of time in state
 - » path to state

Markov Process

- ◆ Assume exponential failure law with failure rate λ .
- ◆ Probability that system failed at $t + \Delta t$, given that it was working at time t is given by

$$1 - e^{-\lambda\Delta t}$$

with

$$e^{-\lambda\Delta t} = 1 + (-\lambda\Delta t) + \frac{(-\lambda\Delta t)^2}{2!} + \dots$$

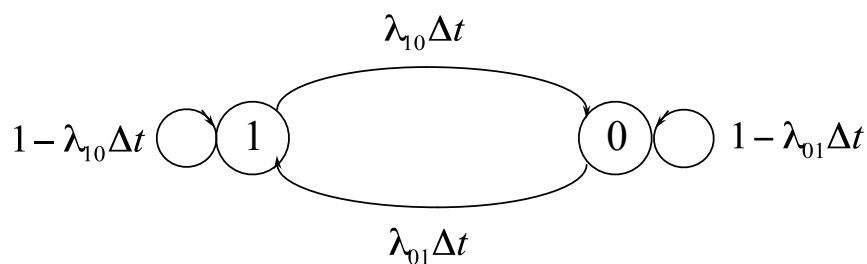
we get

$$\begin{aligned} 1 - e^{-\lambda\Delta t} &= 1 - \left[1 + (-\lambda\Delta t) + \frac{(-\lambda\Delta t)^2}{2!} + \dots \right] \\ &= \lambda\Delta t - \frac{(-\lambda\Delta t)^2}{2!} - \dots \end{aligned}$$

Markov Process

- ◆ For small Δt

$$1 - e^{-\lambda\Delta t} \approx \lambda\Delta t$$



Markov Process

- ◆ Let $P(\text{transition out of state } i \text{ in } \Delta t) =$

$$\sum_{j \neq i} \lambda_{ij} \Delta t$$

- ◆ Mean time to transition (exponential holding times)

$$\frac{1}{\sum_{j \neq i} \lambda_{ij}}$$

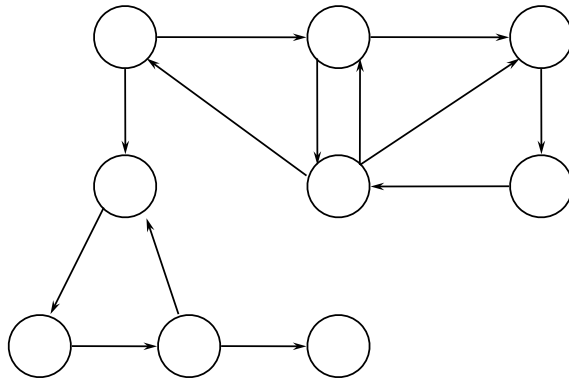
- ◆ If λ 's are not functions of time, i.e. if $\lambda_i \neq f(t)$
 - homogeneous Markov Chain

Markov Process

- ◆ Accessibility
 - state S_i is accessible from state S_j if there is a sequence of transitions from S_j to S_i .
- ◆ Recurrent State
 - state S_i is called recurrent, if S_i can be returned to from any state which is accessible from S_i in one step, i.e. from all immediate neighbor states.
- ◆ Non Recurrent
 - if there exists at least one neighbor with no return path.

Markov Process

◆ sample chain



Which states
are recurrent
or non-recurrent?

Markov Process

◆ Classes of States

- set of states (recurrent) s.t. any state in the class is reachable from any other state in the class.
- note: 2 classes must be disjoint, since a common state would imply that states from both classes are accessible to each other.

◆ Absorbing State

- a state S_i is absorbing iff

$$\sum_{j \neq i} \lambda_{ij} \Delta t = 0$$

Markov Process

- ◆ Irreducible Markov Chain
 - a Markov chain is called irreducible, if the entire system is one class
 - » \Rightarrow there is no absorbing state
 - » \Rightarrow there is no absorbing subgraph, i.e. there is no absorbing subset of states