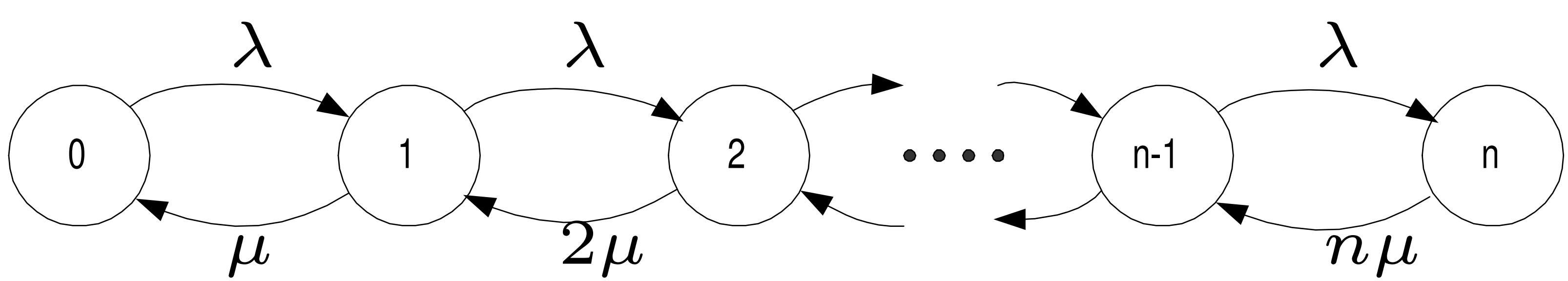
Survivability Quantification

- This section discusses an approach to eliminate the fail-rate in the determination of survivability.
- Source of presentation:
  - *A General Framework for Network Survivability Quantification*,
  - by Yun Liu and Kishor S. Trivedi,
  - in Proceedings of the 12th GI/ITG Conference on Measuring, Modelling and Evaluation of Computer and Communication Systems (MMB) together with 3rd Polish-German Teletraffic Symposium (PGTS), Dresden, Germany, September 2004.
- Application is telecommunication switching system
- The material of the slides are directly drawn from the paper

Survivability Quantification

- Pure Performance Markov Model
  - *n* trunks (channels) with an infinite caller population
  - call arrival process is assumed to be Poisson with rate λ
  - exponentially distributed holding times with rate μ
  - Markov chain shows *i* ongoing calls presented in state *i*
  - what does it mean to be in state *n*: system handling *n* calls, but blocks for all newly arriving calls



Survivability Quantification

- Performance
  - What is the measure of performance?
  - Blocking probability P_{bk}
    - the probability that all n channels are occupied
    - consider steady state probability of being in state j

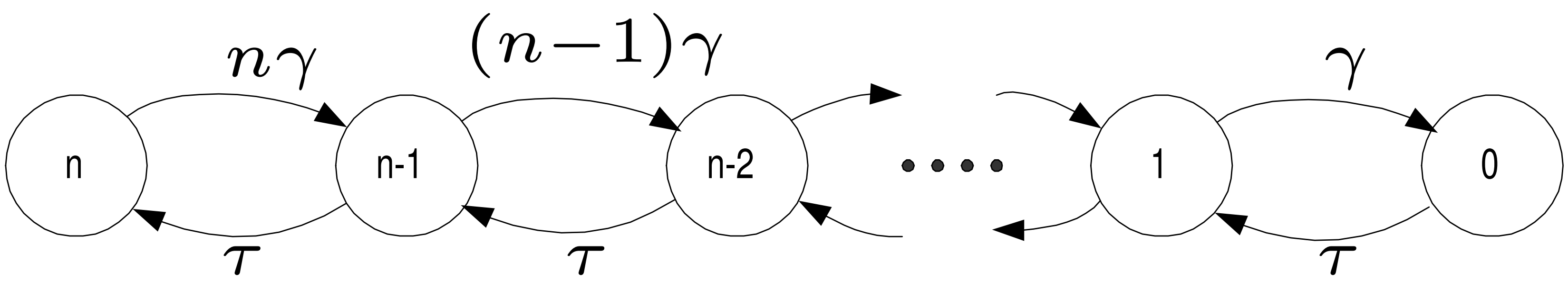
$$\pi_j^P = \frac{(\frac{\lambda}{\mu})^j / j!}{\sum_{k=0}^{n} (\frac{\lambda}{\mu})^k / k!}$$

    - then blocking probability is

$$P_{bk} = \pi_n^P$$

Survivability Quantification

- Pure Availability Markov Model
  - n trunks (channels) with an infinite caller population
  - failure rate γ
  - repair rate τ
  - state i indicates that there are i non-faulty channels in the system
  - what does it mean to be in state 0: system is unavailable



Survivability Quantification

- Availability
  - What is the measure of availability?
  - Steady state probability of state i in pure availability model:

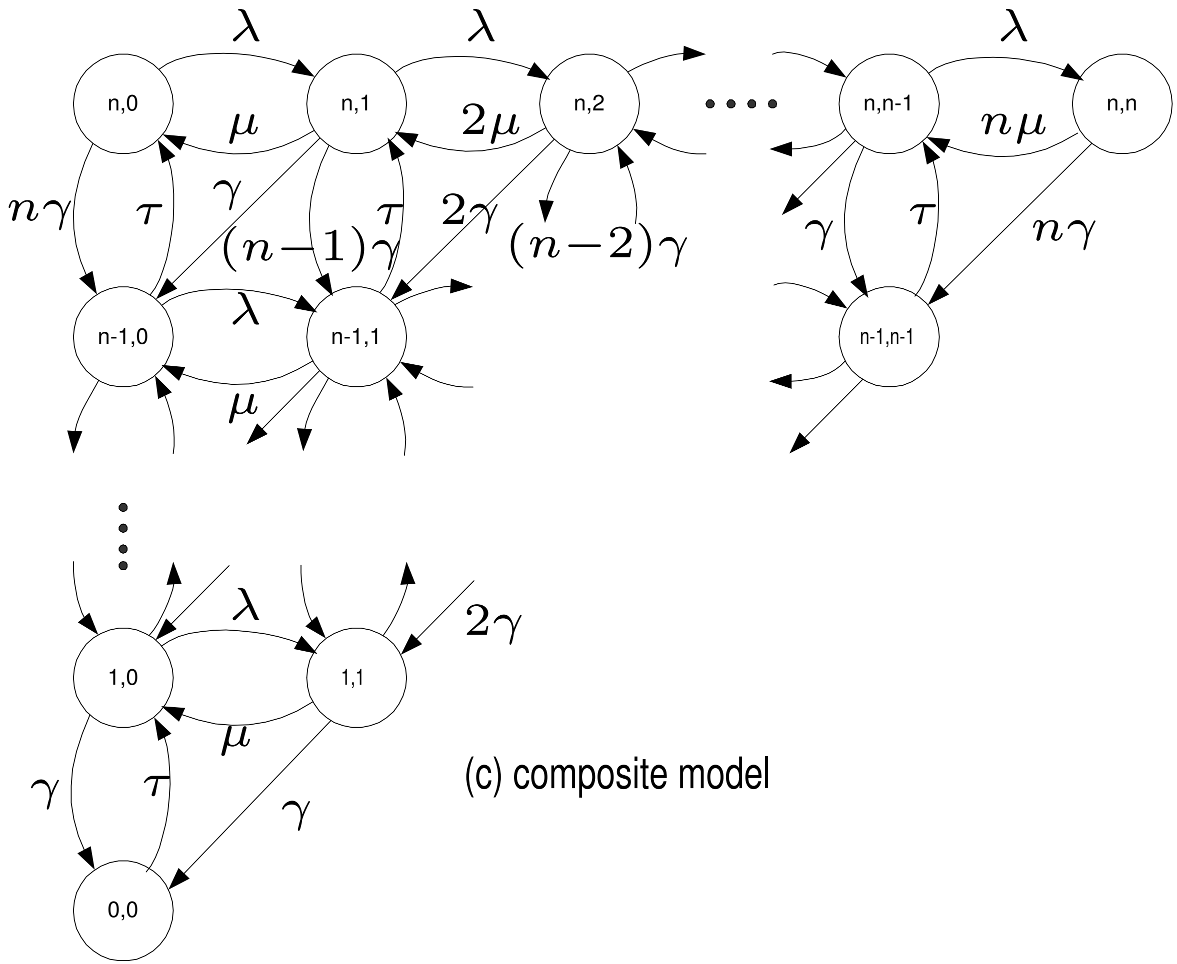
$$\pi_i^A = \frac{\left(\frac{\tau}{\gamma}\right)^i / i!}{\sum_{k=0}^{n} \left(\frac{\tau}{\gamma}\right)^k / k!}$$

  - probability of all channels down is

$$P_A = \pi_0^A$$

Composite Markov Model

State (*i*,*j*) indicates that there are *i* non-failed channels in the system and *j* of them are carrying ongoing calls



(c) composite model

Survivability Quantification

- Performability
  - What is the measure of performance in this combined model?
  - Blocking probability P'_{bk}
    - the probability that all *n* channels are occupied in any “row” of the chain
    - this is the diagonal in the 2-dimensional chain
    - let the steady state probability of state (*k,k*) be denoted by

$$\pi^{C}_{k,\,k}$$

    - then

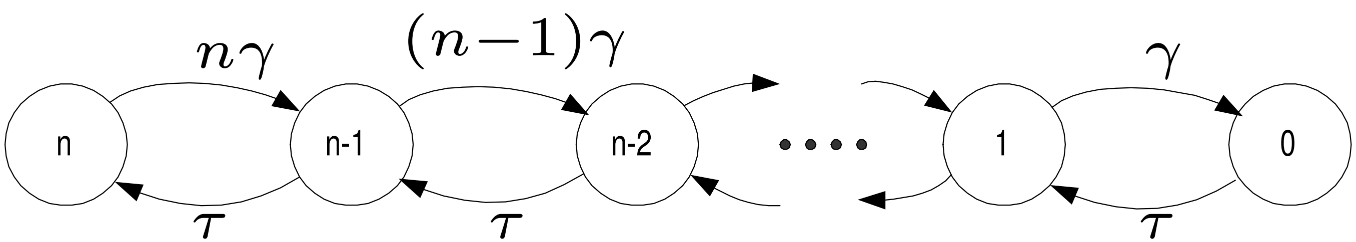
$$P'_{bk} = \sum_{k=0}^{n} \pi^{C}_{k,\,k}$$

Survivability Quantification

- Survivability definition of Knight et.al.
- A survivability specification is a four-tuple, {E, R, P, M } where:
  - E is a statement of the assumed operating environment for the system
  - R is a set of specifications each of which is a complete statement of a tolerable form of service that the system must provide.
  - P is a probability distribution across the set of specifications, R.
  - M is a finite-state machine denoted by the four-tuple {S,s0,V,T} where S is a finite set of states each of which has a unique label which is one of the specifications defined in R; s0 ($s0 \in S$) is the initial or preferred state for the machine; V is a finite set of customer values; T is a state transition matrix.

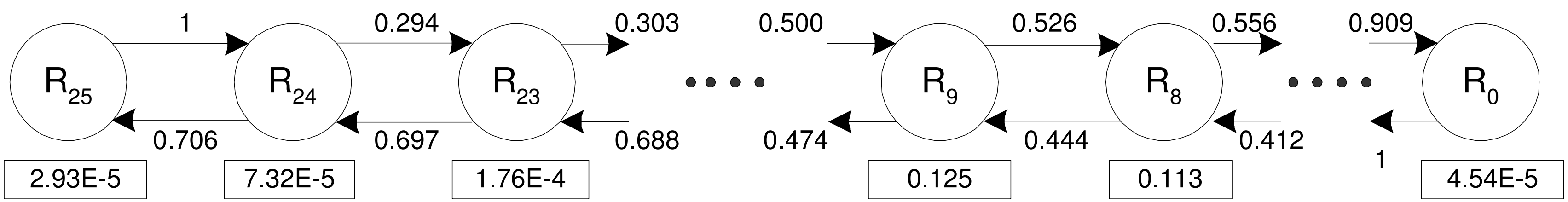
Survivability Specification

- Service specification *R* (*R_n*, …, *R_i*, …, *R_0*)
  – determined by the number of available trunks *i*, *i=n,…,0*



  – what are the transition probabilities from state *i* to *i*+1 or *i*-1

  » $$\frac{i\gamma}{i\gamma+\tau} \qquad \frac{\tau}{i\gamma+\tau}$$

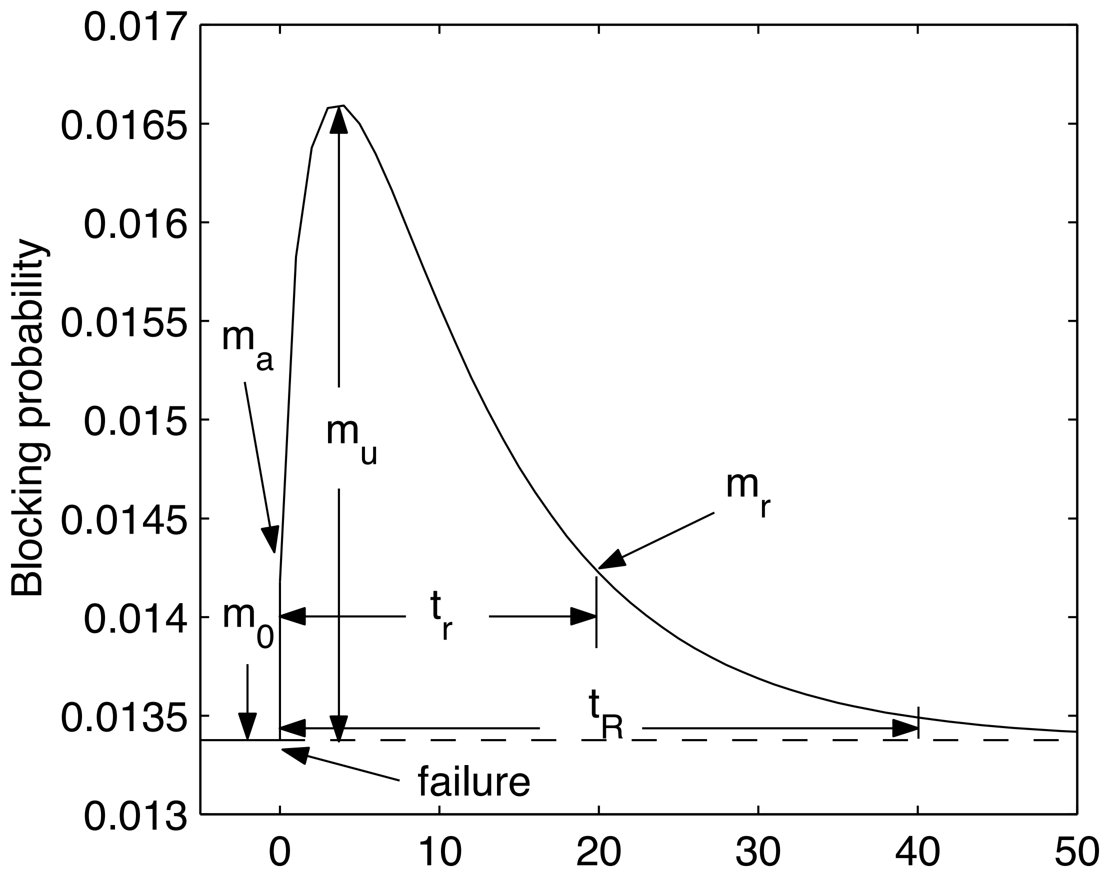


T1A1.2 Model

- Definition 3.
- Suppose a measure of interest M has the value m0 just before a failure happens. The survivability behavior can be depicted by the following attributes: ma is the value of M just after the failure occurs, mu is the maximum difference between the value of M and ma after the failure, mr is the restored value of M after some time tr, and tR is the time for the system to restore the value m0.

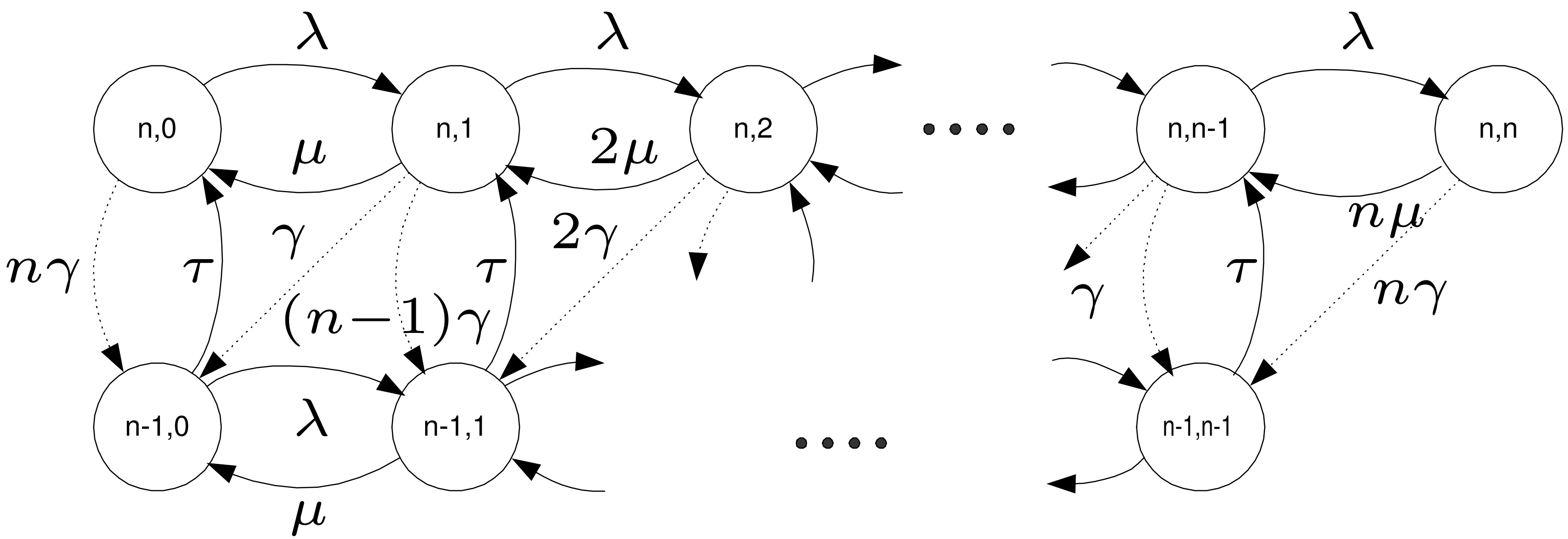
Survivability after 1st Failure

- Based on T1A1.2 definition: Note that it does not matter when the failure occurs.



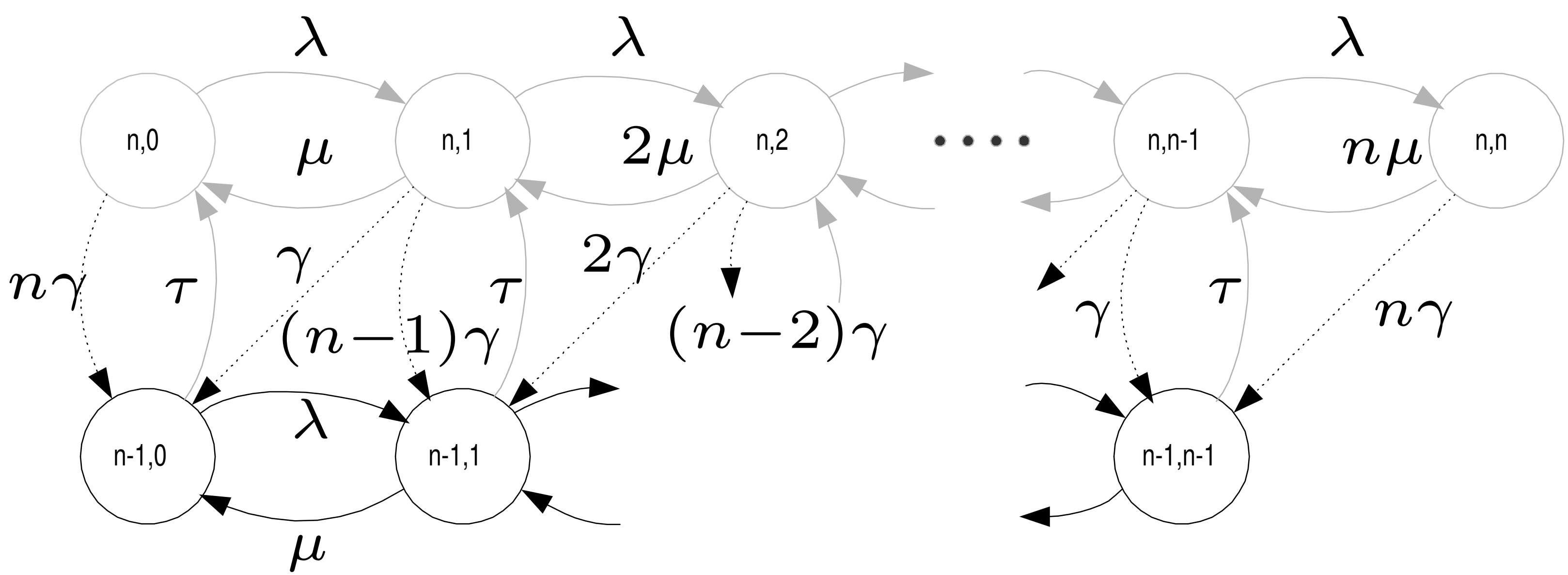
T1A1.2 Markov Model

- Shown is the portion of the previous chain where only the first failure is considered
  - this represents the T1A1.2 model



Truncated composite model

- Model is without repair
  - grey circles and arc represent the removed states and transitions
  - dotted arcs indicate instantaneous transitions have taken place
    - » initial probabilities are from truncated composite model

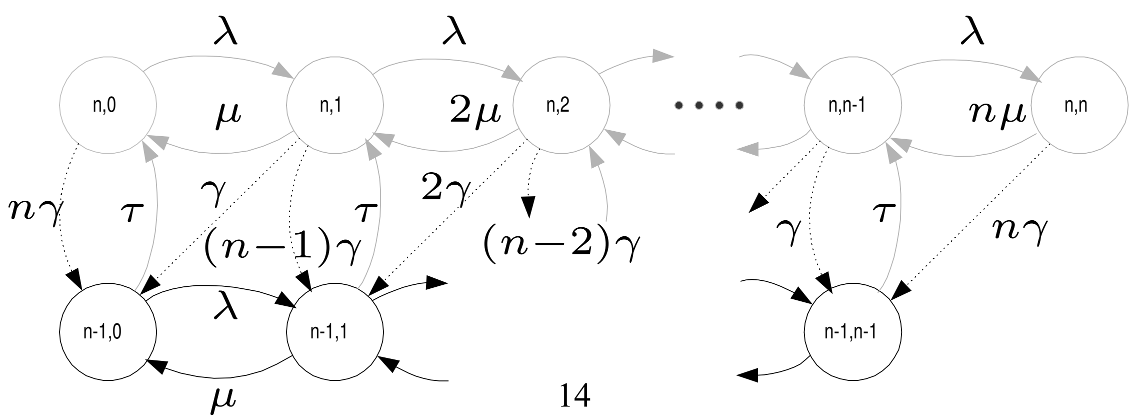


Truncated composite model

- What are the probabilities of begin in a state *i,j* ?
  - since failure has already happened $p^o_{n,j} = 0$
  - and

$$p^o_{n-1,j} = \frac{n-j}{n}\pi^P_j + \frac{j+1}{n}\pi^P_{j+1}$$

  - since state (*n-1,j*) can be reached from
    - » state (*n,j*) with transition rate (*n-j*) γ
    - » state (*n,j+1*) with transition rate (*j*+1) γ
    - » note there is no γ left in this expression! Why?

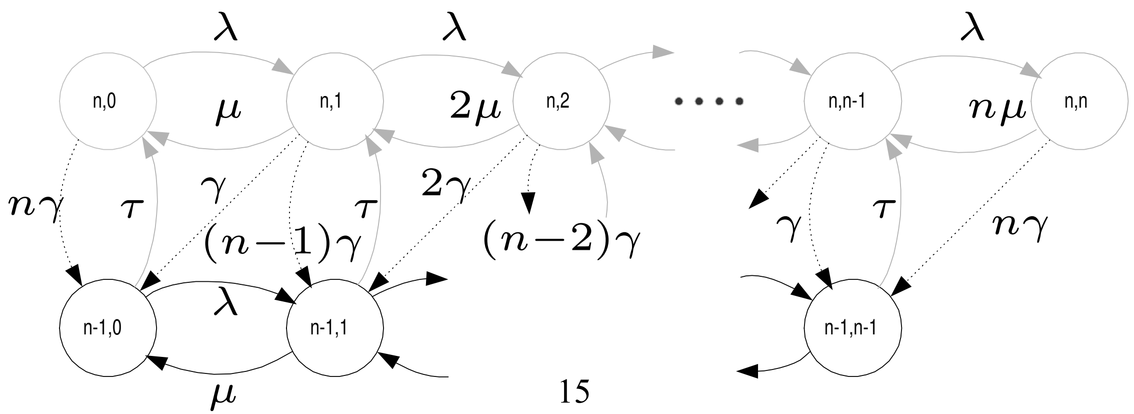


Truncated composite model

- Thus blocking probability is

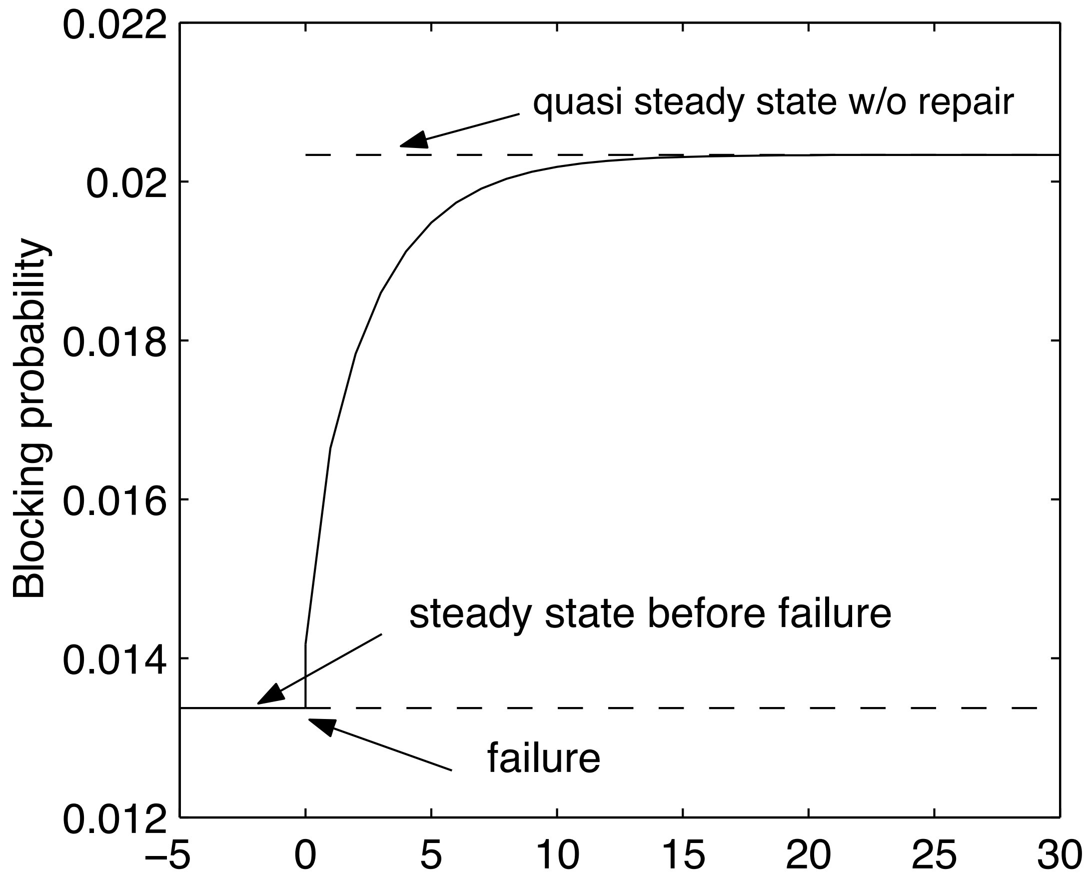
$$P_{bk}(t) = p_{n-1,\,n-1}(t) + p_{n,\,n}(t)$$

  – where $p_{n-1,\,n-1}(t)$ and $p_{n,\,n}(t)$ are the transient probabilities of state $(n-1, n-1)$ and (n, n) in the truncated composite model



Truncated composite model

- Survivability after 1st failure without repair



Survivability Quantification

- The model is then extended to consider more that one (first) faults.
- Note that the approach of the paper overcomes the problems associated with fail-rates, i.e. what is the fail-rate in a survivable system?