Brute Force Strengths and Weaknesses

- Strengths
  - wide applicability
  - simplicity
  - yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)
- Weaknesses
  - rarely yields efficient algorithms
  - some brute-force algorithms are unacceptably slow
  - not as constructive as some other design techniques

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Divide-and-Conquer

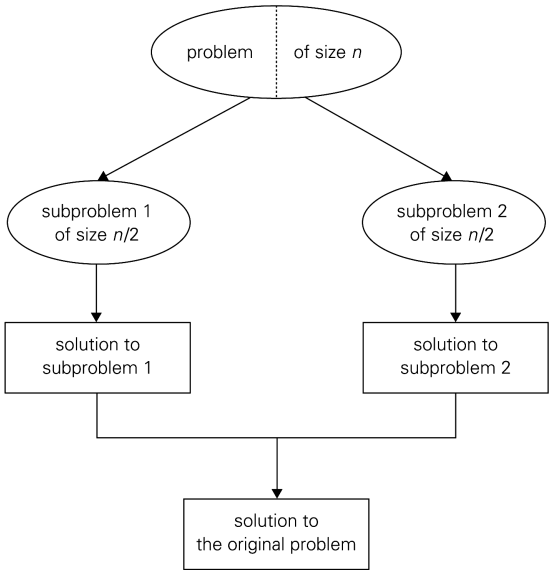

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FIGURE 4.1 Divide-and-conquer technique (typical case)

Divide-and-Conquer: a case for the Master Theorem

Theorem (Master Theorem):

Let $T(n)$ be an eventually nondecreasing function that satisfies the recurrence

$T(n) = aT(n/b) + f(n)$ for $n = b^k$, $k = 1, 2, \ldots$

$T(1) = c$

where $a \geq 1$, $b \geq 2$, $c > 0$. If $f(n) \in \Theta(n^d)$ where $d \geq 0$, then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

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Example: summation

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Mergesort

1) Split array A[0..*n*-1] in two about equal halves and make copies of each half in arrays B and C
2) Sort arrays B and C recursively
3) Merge sorted arrays B and C into array A
   a) copy smallest element from B or C to A
   b) once B or C is processed, copy the remaining unprocessed elements from the other array into A.

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ALGORITHM *Mergesort*(A[0..*n*-1])

//Sorts array A[0..*n*-1] by recursive mergesort
//Input: An array A[0..*n*-1] of orderable elements
//Output: Array A[0..*n*-1] sorted in nondecreasing order

if *n* > 1
copy A[0..⌊*n*/2⌋-1] to B[0..⌊*n*/2⌋-1]
copy A[⌊*n*/2⌋..*n*-1] to C[0.. ⌈*n*/2⌉ -1]

Mergesort(B[0.. ⌊*n*/2⌋ -1])
Mergesort(C[0.. ⌈*n*/2⌉ -1])
Merge(B, C, A)

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ALGORITHM *Merge*(B[0..*p*-1], C[0..*q*-1], A[0..*p*+*p*-1])

//Merges two sorted arrays into one sorted array
//Input: Arrays B[0..*p*-1] and C[0..*q*-1] both sorted
//Output: Sorted array A[0..*p*+*q*-1] of the elements of B and C

i ← 0; *j* ← *0*; *k* ← *0*
while *i* < p **and** *j* < q **do**
    if B[*i*] ≤ C[*j*]
        A[*k*] ← B[*i*]; *i* ← *i* + *1*
    else
        A[*k*] ← C[*j*]; *j* ← *j* + *1*
    k ← *k* + *1*

if i = *p*
    copy C[j..*q*-1] to A[k..*p*+*q*-1]
else
    copy B[i..*p*-1] to A[k..*p*+*q*-1]

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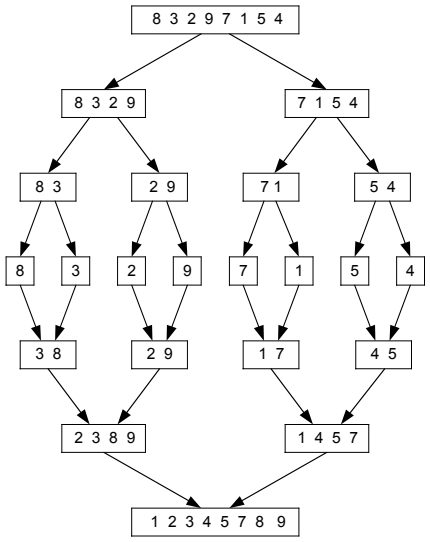
CS395: Analysis of Algorithms

Mergesort



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Analysis of Mergesort

- All cases have same efficiency: $\Theta(n \log n)$
  - Side Note: Number of comparisons in the worst case is close to theoretical minimum for comparison-based sorting:

$$\lceil \log_2 n! \rceil \approx n \log_2 n - 1.44n$$

- Space requirement: $\Theta(n)$
  - version without this requirements exist, but are more costly
- Can be implemented without recursion (bottom-up)

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